Atures of something, I know not what, and innumerable gulls, duck, gannets, etc. We had passed several black swans which suffered us to approach so close that they were taken for moulting which losing their feathers at midsummer cannot fly and are easily taken without firing a shot. This was not the case, however, for the boat nearing them a little more, they rose hissing in the air and moved off.

On our return to the schooner, we were landed on Pelican Island where we found Nanny and Mr. Smith's 4 sheep, the remains of the 12 which I think were here when we visited Muscle Bay before. The soil of the little island is sandy, but rather black looking. It contains probably 3 or 4 acres of land. Some old gum trees, almost denuded of foliage, rise to a considerable height on it. Mr. Smith placed two pair of rabbits on it, which have multiplied to 26. We started several of the young ones and it was a ridiculous thing to see Captain King amongst the rest hallowing and skipping after them. One poor young thing was caught by hand, but though Mr. Smith had begged the gentlemen to kill some if they pleased, it was released. A minor islet called Little Pelican is scarcely separated from the other at low tide.

We had to wait till a late hour for dinner, not wishing to sit down without Mr. Gould and Mr. Smith. I found the latter grown fatter since I saw him last, and equally amiable and contented. He seemed satisfied with his position and so does his wife, who occupies herself with botany, or at least with collecting and preserving flowers. He had almost 8 or 9 police cases brought before him this last whaling season. The punishment is chiefly fines, extending from a sum not less than £2 nor above £20. Their employers are subject to fines not less than £10 nor above £100. He mentioned that one case brought up to him was by a headman or manager who brought up 2 men to be punished because they were cowards, when the whale was seen, they refused to row up to it. Mr. Smith could not enter into a charge such as this. He believes the existence of a police station here has diminished, as it would be expected it might have done, the number of offences committed. They are sent up to Hobarton for punishment on the treadwheel when required. It was suggested that if there were a whalebone breaking establishment here the men under punishment might be more usefully employed.

APPENDIX.

Researches in Relativity

II.—The Basis of the Physical World as indicated by carrying as far as possible the Tenets of Relativity.

BY

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(Read 15th April, 1925)

Additional Errata

In the first paper an unfortunate error of sign occurred early and has necessitated the corrections below. \(\psi\) and \(\omega\) were correctly defined, the first in the last line of page 1 and the second in equation (2) on page 2, but the minus sign is incorrectly put before \(\omega\) on line 24 of page 3, and the following corrigenda are the result:

Page 3, line 24 For \(-\omega\), read \(\omega\)
Page 4, equation (7) For \(-\omega\), read \(-\psi\)
Page 4, equation (10) For \(-\psi\), read \(+\psi\)
Page 5, equation (14) Delete first minus sign.
Page 6, line 17 For
\(-\psi V_1 \Delta \lambda + \psi V_2 \lambda \omega = -\psi V_1 \Delta \lambda + \psi V_2 \lambda \omega\)
read
\(-\psi V_1 \Delta \lambda + \psi V_2 \lambda \omega = -\psi V_1 \Delta \lambda - \psi V_2 \lambda \omega\)
Page 6, equation (15) For \(V_1 \lambda \omega\), read \(-V_1 \lambda \omega\)

I hope this completes the list of necessary corrections.
not even depend on the particular value given to $W$, but are true simply and solely because $W$ and its derivatives are of their respective classes. But this is precisely what the principle of our first paper ensures, though the reader may not be prepared for this statement, and our first task must be to establish it.

It is frequently more convenient to work with the logarithm of such a density as $W$ rather than with $W$ itself. Let $x$ be any log-scalar-density, that is $x$ is of the same nature as $\log W$. We have first to consider how to arrive at invariavtive relations among

$$x, \Delta x, V_\alpha \Delta, V_\alpha \Delta, V_\alpha \beta \Delta, \Delta \tau, \ldots \text{etc.,}$$

where, as usual, $\alpha, \beta, \ldots \text{etc.,}$ are any number of contravariant vector dummies.

Now, from the history of dealing with such continua as we have under consideration, from Riemann to the present day, it may be taken for granted that the problem as so stated necessitates that our continuum must possess structure. (It is open to argument that without such structure we have provided no physical foundation at all.) Riemann provided this structure by the quadratic differential form. Civita-Levi, Weyl, Eddington and Einstein have developed what must be regarded as a generalisation more natural to our present point of view.

The basis of their structure was an intrinsic increment (due to parallel displacement) of vector covariant or contravariant. But from our point of view an intrinsic increment of density (a scalar) is more fundamental. Let us put, in definite forms in parallel, the meanings of these two intrinsic increments. The use of the word "intrinsic" in these two senses implies the following two equations

$$x_a = (D_a - S_a) x = V_\alpha (\Delta x) x$$

$$\delta \tau_a = (D_a - E_a) \delta \tau$$

where the expressions on the left, $S_a, \delta \tau_a$, are absolute increments of the scalar $x$ described above, and of a covariant vector $\delta \tau$. These absolute increments $x_a, \delta \tau_a$, are furnished by comparison with the intrinsic increments $S_a x, E_a \delta \tau$ which are dependent on what may be termed the parallel displacement $\alpha$ (an infinitesimal contravariant vector) and quite independent of choice of coordinates. $D_a$ merely stands for the ordinary differential operator $V_\alpha \Delta$. $E_a$ is a lineity of $\delta \tau$, the lineity itself being linear in $\alpha$; similarly $S_a$ is a lineity of the scalar $x$, the lineity itself being linear in $\alpha$; that is to say $S_a$ is of the form $V_\alpha \delta \tau$, where $\delta \tau$ is a vector. It follows that $E_a, S_a, \delta \tau$, are non-invariantive. Although non-invariantive the relations of these symbols to change of coordinates are quite simple, and it is rather surprising that, so far as I know, they have not hitherto been given in the case of $E_a$.

From the meaning just given to "intrinsic" it follows that $S_a$ is a scalar density and $\delta \tau_a$ is a covariant vector. From this alone follow readily the relations just mentioned of $E_a$ and $\delta \tau$, whether the change of coordinates be finite or infinitesimal. In the present paper infinitesimal change only will be treated of.

The reader will find no difficulty in proving that

$$\delta x = -V_\alpha \Delta x, \delta V_\alpha x = -V_\alpha \Delta, V_\alpha \Delta x$$

$$\delta E_a \delta \tau + \delta \tau E_a = -V_\alpha \gamma \Delta x, V_\alpha \gamma \Delta, V_\alpha \gamma \delta \tau$$

where $\delta'$ and $\gamma$ are the $\delta$ and $\gamma$ used in equations (2), (3), of Art. 15 of W.D.I. (2), in which place infinitesimal change of coordinates was first considered in our notation. (For $\gamma$ see Art. 4 on p. 14 above.)

Note that nothing whatever has been added here to
Art. 7. Alex. McAulay, Relativity II. 24

the meaning (as originally introduced by Weyl and Eddington) of the affine linity \( E_n \). I have in M.D.I. (5) already shown that absolute differentiation is a consequence of the meaning.

Note also some important pure mathematical truisms. Let \( \alpha, \beta, \gamma \) be two scalars of type \( \alpha \); \( \nu, \psi \) vectors of type \( \nu \); and \( E_n, \phi_n \) two affine linities.

Then:
1. \( \alpha - \beta \) is an invariant, \( \nu - \psi \) is a covariant vector, \( E_n - \phi_n \) is covariant, i.e., it is a coexso vector linity whose form is linear in the contravariant vector \( \alpha \).
2. Therefore the general values of \( \alpha, \nu, \phi_n \), are given by
   \[
   \begin{align*}
   \alpha &= \alpha + y, \nu = \nu + \psi, \\
   E_n &= \phi_n + \nu_n
   \end{align*}
   \]
   where \( y \) is an invariant, \( \nu \) a covariant vector, \( \nu_n \) a coexso vector linity which is linear in \( \alpha \). \( \phi_n, \phi \) are in equation (5) any convenient particular functions of their respective types.
3. Both \( \Delta \alpha \) and \( \Delta \phi_n \) are of type \( \nu \). In particular we may take \( \phi_n \) to be either \( \Delta \alpha \) or \( \Delta \phi_n \).

Thus any fundamental scalar density furnishes standard forms for the particular functions \( \alpha, \nu \). The early study of Riemannian Geometry provides a form for \( \phi_n \). Let \( \phi \) be any coexso auto-conjugate vector linity. Then defining \( \phi_\alpha \) by
   \[
   2\phi_\alpha \phi = \Delta \phi \phi
   \]
   we may put \( \phi_n = \phi_\alpha \phi^{-1} \). More particularly, we may define \( \phi \) by saying that \( \phi = 1 \) in some selected system of coordinates, not merely at a single point but at all points.

Art. 8. Alex. McAulay, Relativity II. 25

seems possible at first sight, \( V_2 \Delta \nu \) is covariant. This may be verified at once from equation (5) by putting \( \phi = \Delta \alpha \) for \( V_2 \Delta \phi \) is known to be covariant. It may however be proved by a more familiar process, by summing the absolute increment of any \( \alpha \) round a closed path and observing that as the sum equals the difference of two such values of \( \alpha \) at a single point it must be invariant. Thus \( V_2 \Delta \nu \) is seen to be analogous to the general curvature derivable from the other kind of structure \( E_n \).

Our present quest is for invariant relations as a basis, in our manifold, for a physical world. Can we find such a function \( {\lambda}W \), of the two invariants quantities\( \lambda, W \)

that must exist when structure involves an intrinsic \( \alpha \) and an intrinsic \( \nu \) exists? The necessary and sufficient condition that such a scalar function \( {\lambda}W \) exists was found in our first paper (Art. 3, 6.) to be that

\[
{\lambda}W = \alpha V_2 \Delta \alpha + \nu V_3 \Delta \nu
\]

where

\[
{\lambda} = \frac{\alpha}{W}, \quad W = \frac{\nu}{\alpha}, \quad 1W
\]

As usual \( \alpha \) is quite arbitrary. In a general \( n \)-fold equation (6) imposes \( \pi^2 \) scalar conditions on the \( \frac{1}{2} n(n+1) \) scalar partial first derivatives of \( 1W \) with respect to the same number of independent scalar variables. The number of conditions exceeds the number of scalars at our disposal to satisfy (8). Nevertheless in a four-fold the 16 conditions of (8) are satisfied in one case. For ought we know there may exist a class of such cases, and a four-fold world dependent on the satisfaction of (8) may be possible. The case referred to is when \( 1W = \nu V_3 \alpha^2 \) where \( \nu \) as usual stands for \( \frac{\alpha}{\nu} \), the product of all the primitive units.
Art. 8. Alex. McAvoy, Relativity II.

However this be, such a world would not be that of natural physics. In it there would be no orthogonality or orthodoxy or gravitation. There would only be bulk, inertia, and electric field.

Art. 9. Alex. McAvoy, Relativity II.

law, the conservation of charge \( V_0 \Delta \lambda = 0 \)

It is thus of considerable interest that from our a priori mode of approaching the physical problem neither electric field, nor gravitation, can be supposed absent. Also we may note that ignorance of a mass energy (given by \( \lambda \)) independent of charge seems arbitrary and artificial, for from our standpoint \( \lambda \) seems more fundamental than either \( V_2 \Delta \lambda \) or \( \lambda \).

We shall return to (10) and its connection with the energy tensor later. Meanwhile we resume our a priori approach.

Art. 10. Relativity tenets carried as far as possible.

It is open to argument that the principle "physical laws are independent of choice of coordinates" applies only to the original coordinates \( x_1, x_2, x_3, x_4 \); but it appears more natural to regard the scalars required to specify the structure as co-ordinates to which the principle also applies. Can this be done?

If the physical world is finite in each of its dimensions as held by De Sitter the answer is affirmative. To apply the principle in this its second aspect we have merely to vary these new coordinates, and ensure that the only physical reality namely \( \int \! \! W \) taken over the whole manifold remains unchanged. This is precisely what we did in the first paper (under the name of Stationary Action). If the manifold extends indefinitely in one or more of its dimensions we are not able fully to render \( \int \! \! W \) independent of choice of the new species of coordinates. The breakdown however can be pushed away to an extent that is then possible, and the argument for the naturalness of the process of the first paper retains much of its force.

Here ends our a priori enquiry. Some general aspects of the results of our method will now be considered.

Art. 11. The fundamental identity of relativity.

From the physical side (10) has to be viewed as the stress form (or energy tensor; no longer a "pseudo"
When in (10) we replace \( e \alpha \) by \( \Delta \) the laws take on their vector force form. Note that the electric field and gravitation as well as inertia are included in our meaning of the laws. Indeed a great unification of our ideas of the physical world arises from the straightforward interpretation of (10).

This interpretation was not possible earlier. It required a rearrangement of the foundation stones of general relativity, which was gradually effected by the labours of Weyl, Eddington, and Einstein (see the second sentence of the first paper). Now for the first time we have a complete parallelism of \((e, \omega, \phi)\) with the velocities, and of \((e, \omega, \phi)\) with the momenta, of nineteenth century holonomic dynamics. Hitherto this has not been possible in the case of \( \psi \) and \( \phi \). A formidable obstacle to advance was left in the complexities resulting from the second differential coefficients and the non-linear form of the contracted curvature \( \phi \).

Denote the identically zero form obtained by removing the left-hand member of (10) to the right-hand side by \( jU \); and, putting \( \bar{df} \) for an arbitrary infinitesimal invariant, let the differentials \( dU, \quad d\lambda, \quad \text{etc.} \), be replaced by corresponding fluxes \( \psi dU \), \( \bar{d}\lambda \), where \( dU = \psi d\varepsilon \), \( d\lambda = \bar{d}\varepsilon \).

\( jU \) does not naturally separate into three stresses, but the flux \( dU \) is the sum of three fluxes, kinetic \( dT \), electric \( e \Delta T \), and gravitational \( \bar{e} \Delta T \).

Thus

\[
\psi U = \psi T + \psi T' + \psi T'' = 0
\]

\[
\psi T = \psi V_1 (V_2 e \Delta \lambda - V_1(V_3 e \Delta \omega)\lambda)
\]

\[
\psi T' = \psi V_1 (V_2 e \Delta \omega)\omega - V_1(V_3 e \Delta \omega)\omega
\]

\[
\psi T'' = 2V_1(V_3 e \Delta \omega)\phi - 2V_1(V_3 e \Delta \omega)\phi
\]

\[
(12)
\]

Thus

\[
\psi U = \psi T + \psi T' + \psi T'' = 0
\]

\[
\psi T = \psi V_1 (V_2 e \Delta \lambda - V_1(V_3 e \Delta \omega)\lambda)
\]

\[
\psi T' = \psi V_1 (V_2 e \Delta \omega)\omega - V_1(V_3 e \Delta \omega)\omega
\]

\[
\psi T'' = 2V_1(V_3 e \Delta \omega)\phi - 2V_1(V_3 e \Delta \omega)\phi
\]

\[
(13)
\]

The reader should observe that (12) and (13) follow from the mere assumption \( i\lambda, i\omega, i\psi \) are the first partial derivatives of some scalar density function \( i\tilde{W} \), with respect to the independents \( \lambda, \omega, \psi \). A second form of the assumption is that \( i\lambda, i\omega, i\psi \) are the derivatives of \( i\tilde{W} \) with respect to the independents \( \lambda, \omega, \psi \), where

\[
i\tilde{W} + i\tilde{W} = V_1 (V_2 e \Delta \lambda + V_3 e \Delta \omega + V_4 e \Delta \phi \psi)
\]

(14)

On these results we now superpose those following out of the method, we have based on Einstein's remarkable mathematical discovery. We find that \( i\psi \) gives exactly the expression relativists demand for gravitational force; that \( i\psi \) gives exactly the general electric field of M. B. I. (3), (the allied equation \( V_0 \Delta \omega = 0 \) also following from our method); and that the conservation of energy must exist. To attain this, we have to make the usual assumption that \( i\lambda \) contributes to \( i\tilde{W} \) the one term \( \sqrt{(V_2 e \Delta \lambda)} \lambda \), where

\[
\theta = d\psi^{-1} \cdot [d\psi i\tilde{W} + (\psi - \lambda)]
\]

(15)

The last paragraph asserts the truth of a series of statements which in their entirety may seem a little astonishing or even erroneous. It has been asserted that (15) agrees symbol by symbol with the usually
accepted equation of motion though based on different primary assumptions, and that the associated stress energy tensor is in no sense "pseudo." Why then, it will be asked, is the tensor of the usual theory "pseudo." The explanation is that our present method reveals two new identities which effect the simplification. From the single identity
$$\Delta \Delta U + \Delta \Delta V + \Delta \Delta W = 0$$
the two independent arbitrary values at every point, which involves only \(\lambda, w\), \(d\phi\) and their \(\Delta\) derivatives up to the second order, there arise three independent identities.

The facts about these three were correctly stated in the first paper, but it was not rendered clear why six instead of three do not arise. \(d\phi\) and \(\omega\) may be given independent arbitrary values at every point, while \(\lambda\) is taken to be zero. On now introducing \(\lambda\) the forty-first equation \(V_\alpha \Delta U = 0\), (required to make the integral of \(W\) stationary) appears at first sight inconsistent with the previous forty equations; for \(\lambda\) is expressible in terms of \(\phi\), \(\omega\) and \(E_0\). Thus by a complex indirect way \(\lambda\) is independent on the previously assigned values of \(\phi\), \(E_0\), and the single identity is by no means an identity involving three independent symbols \(\lambda, \phi, \omega\).

Let us now make a somewhat important departure from a usual procedure by supposing \(L\lambda\) to be involved in any way in \(W\), instead of in the very restricted and artificial looking form \(\sqrt{V_\alpha L\lambda}\). On reflection the reader will I believe agree that the non-inclusion equation \(\Delta \Delta U = 0\), and the hydrodynamic term \(d\phi\) in the equation of motion claim our first attention. We use Galilean coordinates and find that, in the non-inclusion equation, \(\sqrt{V_\alpha \Delta U}\) \(\lambda = \lambda_m\) appears as three-dimensional density of mass-energy; and that, in the hydrodynamic equation, for the assumption that as far as \(W\) depends on \(\lambda\) it is

some function of \(\lambda\), the density of matter-inertia appears as \(\lambda_m(\partial / \partial \lambda)W\). If these two three-dimensional densities are identified with each other in the strict mathematical sense, the usual assumption must be made that \(W\) is linear in \(\lambda\). In the immediate neighbourhood of protons and electrons, that is where both densities are to be reckoned in many thousands of tons per c.c. the two must be identified to a very high order of numerical accuracy. Apparently at distances greater than \(10^{-8}\) cm., the densities sink to values comparable with \(10^{-7}\) gm. per c.c. We may well suppose that \(W\) is of the form \(f(\phi, \omega, \lambda, \lambda_m)\), where \(f\) is a finite invariant function of its constituents, for all values of \(\lambda\), inclusive of when \(\lambda\) is indefinitely increased.

It would seem then that we ought to call \(\lambda\) the energy flux and reserve the name momentum vector for \(-\lambda V_\alpha \lambda \phi / \sqrt{V_\alpha \lambda \phi}\).


Extracting from Einstein's illuminating article in "Nature" we have now arrived at a beautifully rounded off relativistic scheme of physics. In direct contrast however to Einstein's concluding words, we appear to have obtained a very promising insight into the true nature of the problem presented by matter, and in what direction to attack the position.

Submitting ourselves with sincere interpretation to the ordinances "do not seek but carry relativistically tense as far as possible", we have found many detailed results harmonising with natural (as opposed to a priori) physics, and not a little which was lacking from former presentations of relativity. Examples are the conservation of energy, the existence of a true energy tensor, and the formulation as an identity of the laws of motion, understood here to include electric field and gravitation. We shall now show that great atomic concentrations of matter and of electric
charge each necessarily presuppose the other. Later, general reasons will be given for expecting such concentrations, and something very like the Bohr orbits accompanying them.

The more I reflect on the dual (matter-electric) aspect of the pair of allied vectors \( \lambda, 1 \lambda \), the more I realise that a long-felt want has here been supplied, a basic natural and simple unification of the three great physical entities matter, electricity, and energy.

Though we may affirm that the dual aspect is prima facie evidence that a rotating mass should be a magnet, the remarks at the end of Art. I1. render it improbable that any numerical deduction is possible from present-day knowledge.

We are on safer, and very interesting, ground when we observe that very high electrostatic potential (irrespective of sign) and very high material density necessarily go together. This, of course, admirably accords with our knowledge of protons and electrons.

Consider the case of a hydrogen atom, where we have very high positive and negative potentials at the proton and electron and an intervening locus at which the potential sinks to zero. This suggests that in our theory we may have to recognize the existence of negative mass. On this point one is inclined at first to argue somewhat as follows. (1) There is no a priori difficulty in supposing mass, either as energy or inertia, to be negative. (2) The total apparent mass of a proton, or of an electron, includes a term due to its charge because of the conservation of energy (though in the absence of such conservation the argument for this electric inertia seems to fail). (3) Observation shows that this total mass is, in each case, positive (for otherwise the two particles would separate), but in the case of the electron the result may be due to the positive electric term masking a negative term contributed by \( 1 \lambda \).

There seems, however, a very real reason preventing us from recognizing negative mass. We seem instead to be impelled to assume that when we reach a point at which \( m = 0 \) we ipso facto reach a boundary of the physical world. In the arguments (1), (2), (3) above, we tacitly assumed that the single scalar condition expressed by saying that the electrostatic potential is zero gives rise to the four scalar conditions expressed by saying that \( \lambda = 0 \). For we assumed that, on each side of the locus where \( m = 0 \), the scalar \( 1 \lambda \) is real. Now, wherever \( m = 0 \) and \( 1 \lambda \) is finite our interpretation of the conditions is that the velocity of light has been attained. The simple view is that this condition holds at the internal boundary of every electron and that in every proton \( m \) attains a very large, or perhaps indefinitely large, value.

The work of earlier writers suggests a first form for \( W \) namely

\[
W = \frac{1}{2} m V_{\alpha}^2 \Phi (V_{\alpha}^2 - 1) \delta \]

where the "extensive" meaning understood for \( d \Phi \) is that which makes

\[
d \Phi = \frac{1}{2} V_{\alpha} \delta \Phi \]

The considerations advanced in the last paragraph suggest a first modification of this form by the addition thereof to \( -l \). The general nature of the Bohr theories suggests a further change by which the invariant coefficient of \( l \) is replaced by a corresponding exponential thus

\[
W = l m - l c h \sqrt{(V_{\alpha}^2 \Phi (V_{\alpha}^2 - 1) \delta)}
\]

Let us inquire whether (16) should lead us to expect the automatic formation of those intense concentrations and the Bohr orbits whose existence has hitherto proved so baffling. Such an enquiry may perhaps suggest further modifications of (16) before we seriously face the labour of exact mathematical analysis. The argument will be easier to follow if we write (16) in the following invariant form

\[
W = l m - l c h \sqrt{V_{\alpha}^2 \Phi (V_{\alpha}^2 - 1) \delta}
\]
Gravitation at once began to make such fortuitous congestions of energy as existed still more congested and to make the emptier places still more empty. Each congestion had a high electric potential and the descending potentials in the emptier surroundings had but one limit namely zero, corresponding to a zero value of energy density. Incipient atoms had evolved from primeval chaos. Each atom consisted of a pair of singular points, at one of which was a concentration of energy and at the other a sink of potential and a boundary of the physical world. Equipartition of energy necessarily ensued, and the incipient atom had become the hydrogen atom with which we are familiar today. Needless to say the details of this brief sketch of the growth of physical law are not to be insisted upon. Rather are they given to indicate in what direction exact analysis is called for.

Similarly (17) while possessing several instructive features bearing on the possible mode of origin of atoms is not very likely to prove the exact form required. If (18) is to be of use in this problem we should expect a general explanation to run somewhat as follows. (1) For a proton $D^2 - H^2$ is positive, and $D$ and $H$ assume large values. (2) For an electron $D^2 - H^2$ is negative and $H$ is between unity and zero. (3) The apparent mass of an electron is practically entirely of electric origin. (4) The energy levels of Bohr's Theory no doubt depend on the periodic cosine term in $W^*$, but the working out of the mathematical details will probably prove difficult.

Near an isolated proton when the electron has been removed it is not improbable that, between limits of distance from the centre of the proton about $10^{-3}$ cm. to $10^{-10}$ cm. $m$ varies approximately inversely as the distance, rising from value unity. When the electron is present it probably pushes a sort of pit or crater of unit density into these denser previously spherical layers, the crater forming a kind of cometary tail,
the electron itself being the head or nucleus. On the other hand it is possible that the critical Rydberg length, about $10^{-5}$ cm., is closely connected with the linear dimension of an isolated electron rather than an isolated proton, and in that case we should expect to attain the value $\gamma$ near an isolated proton, at some distance between $10^{-10}$ and $10^{-9}$ cm.

Why, it may be asked, do not the pairs of concentrations we have pictured run to the extreme of forming one great single pair instead of a vast number of atoms? Four alternative general answers seem reasonable. First, analysis may show that the pairs when once formed will be highly stable. Secondly, the large number of atoms may depend on a constant of integration, perhaps in association with the invariant $\gamma$ of our first paper. Thirdly, a very large mathematical number (such as $10^{16}$) may be involved in the ratio of the linear dimensions of the universe to those of an atom. Fourthly, the bounding vacuities inside electrons may be original unchangeable features of the universe, and form necessary nuclei for the atoms to gather round.