ON THE RELATION BETWEEN THE LOSS OF ENERGY AND THE IONISATION PRODUCED BY CATHODE RAYS.

By J. L. Glasson, B.A., D.Sc.

(Read 14th April, 1913.)

In a previous paper (Phil. Mag., October, 1911) I have shown that the number of ions made by a cathode ray in traversing unit length of air varies inversely as the square of the velocity of the ray. If \( a \) be this number we have the relation

\[
a = \frac{k}{v^2}
\]

where \( k \) is a constant. W. Wilson has shown (Proc. Roy. Soc., vol. 85, p. 240) that the law represented by equation (i) holds also for the \( \beta \) rays given out by radium. These rays had a velocity as high as \( 2.9 \times 10^{10} \) cm. per second. The cathode rays I used had velocities as low as \( 3 \times 10^9 \) cm. per second. So that the law (i) holds over a considerable range of velocities. From data given in my paper, it is easy to calculate the constant \( k \). One cathode ray moving with a speed of \( 4.8 \times 10^9 \) cm. per second makes \( 1.5 \) pairs of ions per cm. of air at a pressure of 1 m.m. of mercury.

So that for air at atmospheric pressure we find that \( a = 1140 \) and \( k = 2.5 \times 10^{28} \).

In a recent paper (Proc. Roy. Soc., April, 1912) Whiddington has shown that when cathode rays pass through
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matter, the velocity of the rays after traversing a distance \( x \) is given by the relation

\[ v_0^4 - v_x^4 = ax \]  

(ii)

and he has given the value of the constant \( a \) for the three substances, aluminium, gold, and air.

By a combination of this result with the result of my experiments it is possible to determine the energy lost by the cathode ray for each ion made by it.

The total number of ions made by a cathode ray in going a distance \( x \) is given by

\[ I = \int_0^x a \, dx \]  

(iii)

Now \( a = \frac{h}{v^2} \), and differentiating (ii) we get

\[ dx = \frac{4v^3}{a} \, dv \]

So that substituting in (iii) and inserting the proper limits the relation becomes

\[ I = \int_{v_0}^{v_x} \frac{h}{v^2} - \frac{4v^3}{a} \cdot dr - \frac{4h}{a} \int_{v_0}^{v_x} v \, dv \]

(iv)

Now if \( E \) is the energy of the ray we have

\[ dE = mvdv \text{ or } v \, dv \]

So that the relation (iv) becomes

\[ I = - \frac{4h}{m} \int \frac{E_x}{E_0} \, dE \]

\[ = \frac{4h}{am} (E_0 - E_x) \]

Let us call the amount of energy lost by the ray for each ion produced \( Q \).

Then

\[ Q = \frac{E_0 - E_x}{I} = \frac{am}{4h} \]

(v)

Since the expression for \( Q \) does not involve \( v \) we see that the ray loses the same amount of energy per ion made whatever its velocity may be. This result is one which might almost have been assumed. In fact, Geiger in dealing with the similar problem in the case of \( \alpha \) rays has made such an
assumption (Proc. Roy. Soc., vol. 83, p. 513). It appears, then, that Whiddington's results (ii) regarding the loss of energy in traversing matter might have been deduced from the law (i) connecting ionisation and velocity, and vice versa.

All the constants in (v) are known, so that we can calculate $Q$.

For air at 760 m.m. pressure Whiddington gives $a = 2.0 \times 10^{-10}$; and $m = 8.8 \times 10^{-28}$ gms., $k = 2.5 \times 10^{-42}$; therefore $Q = 1.76 \times 10^{-10}$.

Expressing this in the usual way as a fall of potential in volts by the relation $eV = \frac{1}{2} mc^2 = Q$ we get $V = 1.1 \times 10^{-2} = 110$ volts. So that for each pair of ions made the ray experiences a loss of velocity corresponding to a fall through 110 volts.

We may compare these results with those obtained for the $\alpha$ rays. Geiger has shown (Proc. Roy. Soc., vol. 84, p. 505) that for the rays from Ra. C. the ionisation per cm. of path varies inversely as the velocity of the rays. From his data I have calculated that the energy lost by the ray per ion made is $5.5 \times 10^{-11}$ ergs. For $\beta$ rays $Q = 1.76 \times 10^{-10}$. So that in making a single ion a $\beta$ ray will lose three times as much energy as an $\alpha$ ray.

There have been many estimates of the energy required to produce an ion, varying from 5 volts up to several hundred volts. Perhaps the most probable value is that given by Townsend, viz., 10 volts. If this value be accepted we see that the proportion of the energy lost by the ray which is actually spent in ionising is fairly small. It seems probable that the bulk of the energy lost by the ray is spent in setting the electrons within the atoms of the gas into vibration insufficient in amplitude to cause their ejection from the atom. This energy of course appears ultimately as heat.

This paragraph is devoted to a consideration of the proportion of the energy of a cathode ray which is spent in ionisation.

Sir J. Thomson has shown (Phil. Mag., April, 1912), that the number of ions made by a cathode ray in traversing unit length of air is given by the expression

$$a = \frac{n\pi e^4}{WT} \quad (vi)$$

where $n$ = number of corpuscles in 1 c.c. of air, $W = $ energy required to ionise an atom, $T = $ kinetic energy of the moving ray ($= \frac{1}{2} mc^2$).
By eliminating \( v \) from (vi) and (i) we get the relation

\[
W = \frac{2n\pi e^4}{km}
\]  

(vii)

Thus from (vii) we may calculate \( W \), the energy of an ion, or by combining (vii) with (v) we get the ratio \( \frac{Q}{W} \), i.e., the energy lost by the ray to the energy spent in ionisation.

Evidently

\[
\frac{Q}{W} = \frac{am^2}{8n\pi e^4}
\]  

(viii)

In order to calculate the value of \( n \) we refer to a paper by Crowther (Proc. Roy. Soc., vol. 84, p. 226), in which he shows that for the five elements C, Al, Cu, Ag, Pt, which have atomic weights varying from 12 to 195, the number of electrons in the atom is three times the atomic weight to within a few per cent. Assuming that this holds also for the atoms of O and N, we find that the number of electrons per c.c. of air at 760 m.m. is \( n = 2.3 \times 10^{21} \).

Putting this value in (ix) we get

\[
\frac{Q}{W} = 5.5 \text{ (approx.)}
\]

Thus the energy spent in ionisation is one-fifth of the whole energy spent by the ray.

I am grateful to Professor Kerr Grant, of the University of Adelaide, for valuable suggestions in connection with this paper.

The University of Tasmania,
29th March, 1913.