

# A MATHEMATICAL MODEL OF A DC MACHINE

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Submitted in fulfillment of the requirements for the degree of  
Master of Engineering Science (Research)



UNIVERSITY  
OF TASMANIA

2004

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Ning Chuang

October 2004

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## **ABSTRACT**

This research aims to determine how a mathematical model may predict the effects of a shorted and/or opened armature coil on a d-c machine. A mathematical model and a simulating computer were developed for a particular 0.375 kW d-c machine under both healthy and faulty conditions.

The model was based on the coupled-coil theory, with a set of first order differential equations that were solved in the time domain. New techniques for measuring inductances on a particular d-c machine were implemented in order to acquire data for development of a simulation. The research found that measurements of armature current waveform, including commutator ripple, agreed quite well with the simulated waveform.

## **Acknowledgement**

During my extremely busy but unforgettable period of study for this research, I have received lots of help and support. I wish to take this opportunity to give my thanks to the following organisations and individuals.

Firstly, I would like to thank the Head and the Deputy Head of the School of Engineering, Professor Frank Bullen and Dr Greg Walker, for their support in my research, so that I could complete this work in such a short time.

Secondly, my sincere thanks should be given to my research supervisor, Dr Richard Langman, and my primary supervisor, Dr Tim Gale. I would like to thank them for their understanding, support and very kind help during the difficult time of my studies. Sometimes I kept them very busy with many issues regarding my work and its progress. I have enjoyed working with Richard in the Power Laboratory in solving some difficult problems with the machine. I also received lots of encouragement and suggestions from Tim. I am so glad that I was able to develop a very close friendship with my supervisors, so that my research project could be kept on the right track to meet the tight deadline.

I would also like to thank Dr Martin Ringrose and Associate Professor Michael Negnevsky, for helping and supporting me while I was learning MATLAB/SIMULINK. I very much appreciate the opportunity that enabled me to start using SIMULINK when I was working for Martin last year.

I also would like to thank a number of the technical support staff in the school, Steve Avery, John McCulloch, Glenn Mayhew, Bernard Chenery, Russell Twining and David Benda. Their help with my research has been very much appreciated.

I wish to give special thanks to a retired lecturer in the School of Engineering, my former university supervisor, Bob Wherrett who has given me support and assistance every time I was faced with various problems.

## *Acknowledgement*

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I am also grateful for Dr Sam Ho's kindness in allowing me to have a copy of the reference papers from his research in 1999. I would also like to thank Prof. Dieter Filbert in Germany for sending me a copy of his published papers.

In the School other postgraduates Andrew Barton, Simon Locke, Andrew Greenhill, Warwick Gillespie, Alan Henderson, Tzuu Bin Ng, Phi Anh Phan, Patrick Burke and Mark Lim have given me varied help and moral support. I often had conversations with them, sharing our research experiences or trying to solve technical problems. I would like to extend them my appreciation.

At the beginning of my research, Hydro Tasmania's Peter Fluckiger helped me by translating a paper in German, and TAFE teacher Rosalind Goodsell translated a French paper. These two papers were very useful to my research, and their many hours' work on technical translations relating to electrical machines has been much appreciated. Here also, my thanks should go to them.

While I was working on MATLAB/SIMULINK, I received assistance from the Applications Engineer from Ceanet, David Willingham, in relation to problems with my model. It may have taken longer to get my model working, without his valuable help, and I would like to thank him as well.

I also need to pass my thanks to the Science Librarians of the University, Chris Evans and Sandy von Allmen, for their very kind help when I was searching my references for this research. It was very hard to find appropriate references in this field, but I found most of them with Chris's and Sandy's help.

I would like to thank SEMF drafter David Belanich for helping me to learn AutoCAD, so that I could draw my many circuit diagrams in CAD within a short time.

Finally, my thanks should be given to my loved dog, Smudge, even though he could not directly help me in my research. But I always found a solution for the problems in my complicated model when I was out walking with Smudge, so I thought he should also be included in these acknowledgements.

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