ERRATA

(1) Page 3, line 2, for $c\lambda$ read $l\lambda$.

(2) Page 5, line 1, delete $\triangle$ before $\log l$.

(3) Page 13, line 17, after $-V_0\sigma_9 l \cup \triangle_9$ add "where $l\cup$ is a co-exco vector linity density.

(4) Page 14,
   (a) Line 11, eq. (24), for cw read cu.
   (b) eq. (25), for $\sum\{d\xi\, cv, cw\}$ and $\sum\{d\xi'cw, cv\}$ read $\sum\{d\xi\, cv, cu\}$ and $\sum\{d\xi'\, cu, cv\}$ respectively.
   (c) line 20 after "referred" insert "to".

(5) Page 16,
   (a) lines 11 to 14 insert commas between pairs $(\xi\xi)$, $(\zeta\zeta)$ etc.
   (b) eq. (30) for $(l_\alpha, l_\alpha{}^{-1})$ read $(l_a, l_a{}^{-1})$ and before $= \sum(i, i^{-1})$ add (e, e)

(6) Page 17, line 19 after "supposed" insert "to be".

(7) Page 19, line 21, for "that" read "than".

(8) Considerable confusion in the references renders the following changes important.
   (a) Page 2, line 7, for (1) read 1
   (b) Page 14, line 20, for (2) read 2
   (c) Page 20, line 17, for (1) read 1, line 23, for (2) read 2.
   (d) Page 20, under "References", for M.D.I. (I), (II), (III), read M.D.I. (1), (2), (3) in all places.

(Added Nov. 1924)

As there have been several months of unavoidable
delay in issuing this first paper, the opportunity is
taken of adding:-

(1)   Some further observations on the calculus used
in this series of researches.

(2)   A statement of the large assistance received
from others that has made publication possible in
face of the inherent difficulties.

(3)   Additions and corrections.

(1)   Our Aims.   As dead as the Sanskrit language.,
Such is the reason given by a referee of the Royal
Society of London why henceforth nothing should ever
be published in Quaternions; and this advice has been
accepted by the Society.   If all mathematicians of
the present day share this view then these papers must
be regarded as addressed to a future generation.

In a popular history of mathematics of about the
year 2000 we may expect to find some such account as
follows:-

"The reader will now realise that the swift advance
of mathematical physics of the last half century has
been largely due to the universal use of the vector
calculus.   It is difficult to explain why the develop-
ment was so long delayed, but we ought to attempt at
least a partial solution.

"The fact seems to be that between 1850 and 1950
nearly all mathematicians failed to make a very nec-
essary distinction.   During those years there were ex-
tant many vector methods and but one vector calculus.
The unique calculus demanded a little preliminary eff-
ort to learn, and the many methods made no such demand.

"About 1890, when the many methods were just begin-
ning to emerge, after a controversy the mathematical
world made its choice — the indolent one.   Between
1920 and 1930 a writer not inaptly described the sit-

uation by paraphrasing the words of the prayer-book
thus: 'They have erred and strayed from the way like
lost sheep; they have followed too much their own
devices; and there is no help in them.'

"What was the predominant feeling of those few pion-
eers who kept on adding to the world's area of reclaim-
ed land though their contemporaries could not be induced
duced to come and inspect it? In such important mat-
-ters as the application of covariants and contravar-
iants to mathematical physics; in proving intergration
theorems of which the meaning remained utterly unknown;
in providing a quite comprehensive and logically sound
dynamical basis for the electromagnetic field; in these
and other respects the pioneers were a full thirty
years in advance of their generation. They would nat-
urally expect to receive some tardy assurance that all
their labours under the sun had not been utterly thrown
away, but instead of this they were informed that
their writings were so useless that for the future
they would not be published. We return to our ques-
tion, what was the mental attitude generated in the
recipients of this uniform ignoration of their work?
Probably they had such meagre satisfaction as results
from a consciousness of having duly performed a nec-
essary thankless task for their fellows. Whether any
fruitful consequences should follow must be left to
the fates to decide."

For the furtherance of our aims, the advance of gen-
eral relativity, the writer believes the means indic-
ated above are far superior to those current to-day.
He therefore deliberately continues to express the re-
sults of his researches in that quaternion form in
which they grew.

(2) Acknowledgements. The past seven months have
been none too long for the preliminary experiments
needed to overcome the many special difficulties of
issue. The work has been truly cooperative.

To Miss M. Clark the writer is beyond measure indebt-
ed for the fact that he is once more acquiring the
beginnings of a mathematical library. She is not a
professed mathematician, but it is extraordinary with
what unerring skill she has been able to follow the
tortuous changes of convention that have occurred in
the writer's difficult task of rendering modern math-
ematics into a braille form.

In April we were still uncertain whether the form
of reproduction the reader sees before him would prove
the most suitable. That it was finally adopted is
mostly due to the ungrudging labour and patent success
of R.G.Brett B.Sc. in the transcription by hand of the
technical mathematical parts as a preliminary to the
work of the photographer and printer. During the past
half year, a period of inefficient knowledge of the
braille character on the part of the writer, the ass-
istance rendered to him by F.M.Young B.A. has been in-
valuable in the matter of the drafting and composition
of the whole paper.

It can scarcely be hoped that even when the mode of
reproduction ceases to be novel we shall ever be cap-
able of freedom from typographical errors whose absen-
ce should be expected were ordinary printing at our
disposal.

(3) Additions and Corrections. The present opportun-
ity is taken of adding to the new notations of Art. 5
eq. (31) another whose want the writer has felt from
as early as 1887. $_\sigma\triangle$, $_q\triangle$, $_\phi\square$ are analogous symbols
whose pre-suffix is a very inconvenient symbol to
print, especially if of any complexity such as when

$$\phi = V_1(\ )\omega$$

I have recently thought of a mode of dealing with
these worth adopting permanently; and shall henceforth
freely use the notation about to be described. Put

$$_q\triangle = \%_0 q, \quad _\phi\square = \%_0 \phi \qquad (A)$$

The symbol $\%$ should always be read as "rate - per"
and should never occur without its immediately follow-
ing independent variable (the q or ϕ of (A).) Thus
it is never to be used in place of $\triangle$; the new equiv-
alent of $\triangle$ is $\%\rho$, though doubtless it will prove
convenient to retain $\triangle$ as an alternative of $\%\rho$.
The sign to be attached to $\%$ is given thus: let
X be any function of the independent q or ϕ .
Then without exception the following is to be true.

$$\left.\begin{array}{l} dX = V_0 dq \% q \cdot X , \\ dX = V_0 d\phi e \% \phi e \cdot X \end{array}\right\} \qquad (B)$$

It may be noted that $\%$ is the analogue of ∂/∂
in $\left(\partial/\partial x\right)$.