AN ORIGINAL AND INDEPENDENT METHOD OF EXTRACTING ROOTS OF NUMBERS.

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Let us look at these three formulæ:—

$$\frac{1}{R}\left\{(R-1)\,N+\frac{\mathbf{N}}{N^{R-1}}\right\} \quad \ldots\ldots\ldots \quad A$$

$$\left\{N^{R-1}\times\frac{\mathbf{N}}{N^{R-1}}\right\}^{\frac{1}{R}} \quad \ldots\ldots\ldots \quad B$$

$$\frac{1}{R}\left(N-\frac{\mathbf{N}}{N^{R-1}}\right) \quad \ldots\ldots\ldots \quad C$$

Where R is the degree of the root sought, $\mathbf{N}$ is the number whose root is required and N is conveniently the nearest known number greater than the root.

Let us refer to them as A, B, and C respectively to save space.

A is the arithmetical mean of the same quantities of which B is the geometrical mean, and therefore A is greater than B, but B is the root we require; A is also less than N because a mean is less than the greater of the extremes, therefore A is nearer the root than N.

By using the formula A again, putting the value of A in the place of N, we get another value for A still nearer the root and so on *ad infinitum.*

The difference between N and A is C, which does not vanish, while N is not the root, so that we by repeating the process can get a value for A as near the root as we please.

Further! We always know whether we have chosen our value for N correctly, i.e., greater than the root, because otherwise the value of C becomes negative.

Let us take the following example of the method, where the labour of the work is also shewn to be capable of considerable reduction by cutting off end figures, etc.

To get the fifth root of seventeen. Taking 2, which is evidently too great to start with, we have by formula A

$$\frac{1}{5}\left\{4\times 2+\frac{17}{2^4}\right\}=\frac{29}{16}$$

Taking off 2 from the numerator and 1 from the denominator slightly lessens the value of the above fraction, but makes it more workable by reducing it to $\frac{9}{5}$, which we can again use in the formula A in decimal form, thus,—

$$\frac{1}{5}\left\{4\times 1{\cdot}8+\frac{17}{(1{\cdot}8)^4}\right\}$$

$$=\frac{1}{5}\left\{7{\cdot}2+\frac{17}{10{\cdot}4976}\right\}$$

We can alter this to

$$\frac{1}{5}\left\{7{\cdot}2+\frac{17}{10{\cdot}5}\right\}$$

without much risk of getting a value too small, and then this is equal to

1·7638 . . . . .

Let us make a trial with

1·763

If it should be too small the next number will be greater instead of less, because by the general difference formula C, the difference will be negative.

With this number, formula A gives us,

$$\frac{1}{5}\left\{4\times 1{\cdot}763+\frac{17}{(1{\cdot}763)^4}\right\}$$

$$=1{\cdot}76234084128\ \ldots\ldots$$

Correct to six places of decimals and perhaps more, as shown by noticing the decimal place of the first significant figure in the last difference, i.e.,

$$\begin{array}{l} 1{\cdot}763 \\ 1{\cdot}76234084128\ \ldots \\ \hline 1{\cdot}00065915871\ \ldots \end{array}$$

In general we can rely upon twice the number of places of decimals as the decimal place of the last cipher in the last difference, which in this case is 3.

The total value of A is too great as all values found this way must be, but the number above with no more figures may even be too small; as a rule, if we stop short according the rule stated, we get a value too small.

In reality the value is correct up to the first place, which makes it too great.