

MODAL EXPANSION THEORIES FOR
SINGLY - PERIODIC DIFFRACTION GRATINGS

by

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John R. Andrewartha
August, 1981.

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SUMMARY

This thesis reports on the analysis and development of rigorous modal expansion techniques for determining the scattering properties of singly-periodic diffraction gratings. Both reflection and transmission gratings are considered, and although emphasis is given to a theoretical study of the formalisms, many numerical results obtained with the latter are also presented. Most of the formalisms pertain to gratings having specific groove geometries and infinitely-conducting surfaces. However, in two cases one or other of these constraints is removed.

Several of the established formalisms, based on a variety of non-modal techniques, are reviewed, and the essence of their method described. The advantages that modal treatments have over these methods are explained, and previous applications of the former are summarised.

Initial theoretical investigations concern the rectangular-groove grating. Intensive studies reveal an alternative approach to the concept of diffraction resonance anomalies. They also provide new insight into the understanding of this grating's overall behaviour, including its blazing and passing-off properties in the first-order Littrow mounting. These ideas are usefully extended throughout the thesis to encompass the behaviour of all gratings.

The theoretical treatment for the rectangular-groove grating is adapted to account for the diffraction properties of three unusual profiles which also possess a rectangular geometry. Two of these structures consist of a transmission grating on a reflecting element, and are shown to exhibit a pronounced resonance action. Tuning of the various grating parameters governs the behaviour of the resonances and indicates the potential use of these devices as a type of reflecting Fabry-Perot interferometer. The third structure is a stepped reflection

grating which proves capable of accurately modelling the performance of general profile gratings including those with sinusoidal and triangular profiles.

Single and bi-modal expansions are shown to provide useful field approximations for not only the conventional rectangular-groove grating, but also for two of the three related structures. These approximations aid in the examination of resonances and other spectral phenomena. Their regions of accuracy and validity are determined.

The assumption of perfect conductivity is relaxed in a formalism which is described for dielectric and lossy metallic surfaces. The method is tailored specifically to the rectangular-groove profile and is one of the few modal expansion techniques appropriate to non-perfectly conducting gratings.

The thesis concludes with the presentation of two formalisms which employ an impedance-related condition to completely specify a set of modal functions. The first formalism prescribes a solution for a grating whose grooves are semi-circular in cross-section. Eigenfunctions for a circular waveguide constitute the modal functions. The second formalism accommodates reflection profiles of general groove cross-section, and utilizes a superposition of the rectangular-waveguide eigenfunctions. The first formalism is employed to evaluate in detail the spectral performance of the semi-circular groove grating, while the second is applied, not only to this grating, but also to the triangular and sinusoidal groove gratings.

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