REASONING ABOUT COVARIATION WITH TINKERPLOTS

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Declaration of Originality

This thesis contains no material which has been accepted for a degree or diploma by the University or any other institution, except by way of background information and duly acknowledged in the thesis, and to the best of my knowledge and belief no material previously published or written by another person except where due acknowledgement is made in the text of the thesis, nor does the thesis contain any material that infringes copyright.

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Statement of Ethical Conduct

The research associated with this thesis abides by the international and Australian codes on human experimentation, as set out in the National Statement of Ethical Conduct in Human Research (2007) and interpreted by the Human Ethics Committee of the University.

Ethical approval for the research was gained from the Southern Tasmania Social Sciences Human Research Ethics Committee at the University of Tasmania in 2006 – Ethics Approval Number H8778. The committee adheres to the guidelines outlined in the National Statement on Ethical Conduct in Human Research. The research also had permission and approval from the Department of Education, Tasmania, and satisfied department criteria for Conducting Research in Tasmanian Government Schools and Colleges (2006).

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Statement of Co-authorship

The following people and institutions contributed to the publication of the work undertaken as part of this thesis:

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Noleine Fitzallen contributed the Historical Developments in Graphing and the Current Graphing Curricula sections. She also contributed to the refinement and presentation of the report. The report is independent of the thesis and does not include any data used in the thesis. The Historical Developments in Graphing section is reproduced in part in this thesis. The authorship of other information from the report included in the thesis is duly acknowledged.

We the undersigned agree with the above stated “proportion of work undertaken” for each of the above published (or submitted) peer-reviewed manuscripts contributing to this thesis:

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My life’s journey has taken many twists and turns, providing me with the opportunity to experience a number of different careers and meet a diverse range of people. My partner once quipped “You haven’t made up your mind what you want to be!” This may be the case but to me it does not matter. I want to explore new things, places and ideas. I want to continue to grow and develop and embrace new opportunities. This thesis is a reflection of my personal experiences and achievements and is influenced heavily by my family, people I have worked with and people I have not met, yet know through their research. Some people assisted greatly in the production of the actual artefact but many more had an impact on why I was in this place, at this time, to be able to embrace the opportunity of conducting research when it was offered. I would like to acknowledge that this thesis is a culmination of all these influences.

The story in this thesis is about 12 students. They graciously let me into their lives to extend, question, and examine their thinking. It was a very rewarding experience both professionally and personally. I am extremely grateful for the opportunity to work with them. My personal thanks go to Blaire, Jake, James, Jessica, Johnty, Kimberley, Mitchell, Natalie, Natasha, Rory, Shaun, and William.

Special thanks are extended to my supervisors Jane Watson and Natalie Brown. Among other things, Jane allowed me the freedom to be creative, found the time to debate points of view to extend my thinking, and had the patience to assist me to improve my writing. Natalie valued my ideas, challenged me to think in different ways, and supported me to publish my work.

I must also thank the designers of TinkerPlots, Cliff Konold and Craig Miller. The creative and innovative approach taken to develop TinkerPlots has resulted in the
development of innovative data analysis software that has these features imbedded in its design. Consequently, students using *TinkerPlots* are empowered to be creative and innovative in return.

This research was funded by an Australian Postgraduate Award – Industry Scholarship associated with the Australian Research Council Linkage Project LP0560543 and the industry partner, Department of Education, Tasmania (DoET). The linkage project, *Providing the Mathematical Foundation for an Innovative Australia within Reform Based Learning Environments* (MARBLE) (Watson, Beswick, & Brown, 2011), provided the context for this thesis.
Covariation is recognised as an important aspect of statistical thinking and reasoning and is used to explore the relationship between two attributes. Often, covariation is determined from the interpretation of scatterplots that display the correspondence of two numerical attributes and is described as a trend in the data. Scatterplots are utilised when conducting exploratory data analysis (EDA). EDA strategies are useful for interpreting the data as they allow the data to be manipulated in order to construct graphical representations that facilitate making sense of the data. The translation of EDA strategies into innovative software packages, such as TinkerPlots: Dynamic Data Exploration, has placed student learning about data analysis in technological environments and there is a need to investigate the way in which students learn in these contexts.

This inquiry had two objectives. The first objective was to further understanding of the factors that influence student learning when working with software packages. This is through the development of a conceptual framework for learning in EDA graphing environments that aligns with and extends current research about student understanding of graphing and data analysis. The second objective was to explore the intersection between the students’ thinking and reasoning about covariation and the influence of TinkerPlots on that process, as students explore data sets to determine the relationship between variables and identify trends. To realise these objectives the following research questions are explored:

1. How can the learning behaviours of students as they engage with exploratory data analysis software be characterised through a framework that can then be used to explore and analyse students’ understanding of covariation using TinkerPlots?

2. How do students interact with the exploratory data analysis software, TinkerPlots, to represent data in a variety of forms when exploring questions about relationships within a data set?
3. How do students demonstrate an understanding of covariation in the exploratory data analysis software environment afforded by TinkerPlots and use these understandings to provide informal justification for their conclusions about the relationships identified?

The inquiry employed an educational design research methodology within a pragmatist paradigm to facilitate the development of a systematic iterative study. The methodology was chosen to encapsulate the way students learn about the interpretation of graphical representations, more specifically related to covariation, in the technological software environment afforded by TinkerPlots.

The inquiry was enacted across seven stages. Stage 0 involved the development of the research design. Stage 1 involved the development of a conceptual framework for learning in EDA software environments that incorporated four aspects of graphing and data analysis skills – Generic knowledge, Being creative with data, Understanding data, and Thinking about data. Stage 2 involved an evaluation of TinkerPlots to determine its usability as a teaching and learning tool. Stage 3 involved the development and evaluation of an assessment tool to determine the prior learning of students in relation to the interpretation of graphs, and select the participants for the data collection stage of the inquiry. Stage 4 involved the development and implementation of a sequence of learning experiences. The activities in the sequence of learning were based on recommendations from the research on the development of graphing and data analysis skills. The sequence of learning experiences was implemented with 12 students working in pairs, twice a week for 45 minutes, over a period of 6 weeks. In addition, the data generated from individual interviews with the 12 students conducted at the end of the sequence of learning were included in this stage. The data from the student interviews are presented as Student Profiles that encapsulate the way in which they used TinkerPlots to develop not only an understanding of covariation but also develop other data analysis skills and strategies. Stage 5 involved the presentation of the results for the Research Questions, with the discussion of the findings, implications of the inquiry, and recommendations for future research included in Stage 6. The presentation of the thesis follows this chronological order.
Through the evaluation of *TinkerPlots* and its subsequent implementation in the inquiry, it was identified that *TinkerPlots* provides a powerful learning environment for supporting students’ understanding of covariation. In terms of student understanding of covariation, the inquiry identified that young students are able to reason about covariation and display three levels of reasoning. The results also suggest that students adopt three different strategies when accessing the features of *TinkerPlots* while creating and interpreting graphs. These strategies are: *Snatch and Grab, Proceed and Falter, and Explore and Complete*.

Outcomes of the inquiry are presented in relation to the thesis-developed *Model of Learning Behaviour in EDA Graphing Environments*. Within the framework of the model the students’ development of covariation reasoning is revealed and discussed in terms of the potential of the results to inform the teaching and learning of covariation within EDA software environments and future curriculum development. Consideration was also given to the merits of the *Model of Learning Behaviour in EDA Graphing Environments* and its application throughout the inquiry process. Unexpected insights into the students’ thinking and reasoning about association are also discussed to demonstrate the utility of the thesis-developed model and to highlight the need to further research in the area of student understanding of association.
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