Developing Students' Understandings and Representations of Statistical Covariation

by

Jonathan Moritz

(B.A., B.Sc., Dip. Ed.)

Submitted in fulfilment of the requirements for the Degree of Doctor of Education

University of Tasmania

(November, 2006)
Declaration of originality

This thesis contains no material which has been accepted for a degree or diploma by the University or any other institution.

No material in this thesis has been previously published or written by another person except where due acknowledgement is made in the text of the thesis, nor does the thesis contain any material that infringes copyright. Appendix 2 presents material previously published with content related to this thesis.

Signed: ______________________________

Date: ______________________________
Statement of authority of access

This thesis may be made available for loan and limited copying in accordance with the Copyright Act 1968.

Signed: ______________________________

Date: ______________________________
Abstract

*Statistical covariation* refers to the correspondence of variation of two statistical measures that vary along numerical scales. Reasoning about covariation commonly involves translation processes among three representations: (1) numerical data, (2) graphical representations, and (3) verbal statements such as “taller people tend to be heavier.” Two well-known translations are *graph production* and *graph interpretation*. Less well known is the process of *speculative data generation*, involving translating a verbal statement into a possible graph or other data representation. This study explored school students’ reasoning involving these three translation skills through various tasks in surveys and interviews. Evidence is presented concerning methods to assess these skills, and concerning how students as young as third-grade can engage covariation tasks involving familiar contexts. Interviews involved prompting for cognitive conflict using responses from other students, and provided evidence of limited engagement of ideas that were slightly more sophisticated than their own responses.

Responses for each of the three translation skills were described within assessment frameworks involving four levels – Nonstatistical, Single Statistical Aspect, Inadequate Covariation, and Appropriate Covariation – distinguished by the structure of combining correspondence and variation. Distinguishing features of the levels suggested stages of development that may inform instruction. For development from prior beliefs to data-based judgements, tasks involving counterintuitive covariation were designed to prompt students to engage data. For development from single variables to bivariate data, time was observed as a natural covariate, implicit in statements such as “it’s getting hotter,” with a connotation of order that supported pattern recognition of passing time being associated with
corresponding change in a measured variable. For development from single cases to
global trends, many students represented correspondence in a single pair of values, at
the expense of representing variation. Tasks involving discrete data with few cases,
and the use of case labels in responses, were observed to support the view of two
data values each linked to the same corresponding case label. This consolidated view
of correspondence supported consideration of additional bivariate cases involving
variation. Students tended to articulate covariation using the language of comparison
and change.

Findings were related to issues in the historical development of coordinate
graphing, to findings from educational research in statistics, algebra, science and
psychology, and to recommendations within curriculum documents. Student
representations of statistical covariation were observed to provide a window into
statistical reasoning, and are advocated as a valuable basis for classroom discussions
to help develop statistical literacy.
I would like to acknowledge my supervisor, Dr Jane Watson, University of Tasmania, for her support over many years. Her guidance in my research career, and particularly during my candidature, encouraged inquiry, rigour, and commitment. Her balance of nudging and patience were amazing. I would also like to acknowledge the examiners who provided helpful suggestions for amendments.

I would like to acknowledge my family, for their nudging and patience. They made personal sacrifices throughout my candidature, including listening to lengthy ramblings, accommodating volumes of paperwork, and permitting the tapping of keystrokes during all hours.
# TABLE OF CONTENTS

## CHAPTER 1. INTRODUCTION

1. **OVERVIEW OF CHAPTER 1** .................................................. 1.1
2. **STATISTICAL COVARIATION** ............................................. 1.1
3. **UNDERSTANDINGS AND REPRESENTATIONS OF STATISTICAL COVARIATION** ................................................................. 1.3
4. **THE SIGNIFICANCE OF THE TOPIC** .................................. 1.6
5. **THE STRUCTURE OF THE STUDY** ...................................... 1.7
6. **THE EVOLUTION OF THE STUDY** ...................................... 1.9

## CHAPTER 2. LITERATURE REVIEW

1. **HISTORY OF GRAPHING AND COVARIATION** ..................... 2.1
   1.01 Development of Coordinates and Functions ..................... 2.3
   1.02 Empirical Data of the 17th and 18th Centuries .................. 2.4
   1.03 Playfair’s Works ...................................................... 2.5
   1.04 Increasing Recognition of the Power of Graphics ............ 2.7
   1.05 Contemporary Uses of Graphs ..................................... 2.8
   1.06 Historical Implications ............................................ 2.12
2. **SCHOOL CURRICULUM** .................................................. 2.13
   2.01 The Broad Context for Statistics Education ................. 2.13
   2.02 Covariation in School Curricula ................................ 2.15
   2.03 Algebra in School Curricula ...................................... 2.16
   2.04 Data Handling and Graphing in School Curricula .......... 2.17
3. **RESEARCH LITERATURE** ................................................. 2.21
   3.01 Previous Research Reviews and Perceived Gaps ............. 2.21
   3.02 The Structure of the Review of Research Literature ....... 2.24
   3.03 Graph Interpretation ............................................... 2.24
   3.04 Covariation Interpretation ....................................... 2.29
   3.05 Graph Production .................................................... 2.37
   3.06 Speculative Data Generation ...................................... 2.43
   3.07 Summary of Research and Future Directions .................. 2.47
   3.08 Prior Research Overlapping this Thesis ....................... 2.48
   3.09 Summary of Key Terms ............................................. 2.49
## CHAPTER 3. RESEARCH DESIGN AND METHODOLOGY

### 3.01 SUMMARY

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.01</td>
<td>3.1</td>
</tr>
</tbody>
</table>

### 3.02 RESEARCH AIMS AND EXPECTATIONS

<table>
<thead>
<tr>
<th>Aim</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.02.01 Aim 1 – Assessment Frameworks of Student Responses</td>
<td>3.2</td>
</tr>
<tr>
<td>3.02.02 Aim 2 – Evidence of Student Performance</td>
<td>3.3</td>
</tr>
<tr>
<td>3.02.03 Aim 3 – Reasoning and Reactions to Conflicting Responses</td>
<td>3.4</td>
</tr>
<tr>
<td>3.02.04 Aim 4 – Conceptual Development across Skills</td>
<td>3.4</td>
</tr>
</tbody>
</table>

### 3.03 THEORETICAL ORIENTATION

<table>
<thead>
<tr>
<th>Theoretical Orientation</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.03.01 Theoretical Developmental Model for Assessment</td>
<td>3.5</td>
</tr>
<tr>
<td>3.03.02 Misconceptions and Cognitive Conflict</td>
<td>3.7</td>
</tr>
</tbody>
</table>

### 3.04 DIMENSIONS OF RESEARCH METHODS OF STUDENT ASSESSMENT

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.04.01 Issues in Task Design for Paper-based Questions</td>
<td>3.12</td>
</tr>
<tr>
<td>3.04.02 Issues in Task Design for Interviews</td>
<td>3.13</td>
</tr>
</tbody>
</table>

### 3.05 DEVELOPING TASKS

<table>
<thead>
<tr>
<th>Task Development</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.05.01 Task Development – Data Collection 1</td>
<td>3.20</td>
</tr>
<tr>
<td>3.05.02 Task Development – Data Collection 2</td>
<td>3.23</td>
</tr>
<tr>
<td>3.05.03 Task Development – Data Collection 3</td>
<td>3.24</td>
</tr>
<tr>
<td>3.05.04 Survey Tasks – Data Collection 3</td>
<td>3.27</td>
</tr>
<tr>
<td>3.05.05 Interview Tasks – Data Collection 4</td>
<td>3.35</td>
</tr>
</tbody>
</table>

### 3.06 DESIGN AND PROCEDURES

<table>
<thead>
<tr>
<th>Design and Procedures</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.06.01 Participants and Procedures Common Across Data Collections</td>
<td>3.35</td>
</tr>
<tr>
<td>3.06.02 Data Collection 1</td>
<td>3.37</td>
</tr>
<tr>
<td>3.06.03 Data Collection 2</td>
<td>3.40</td>
</tr>
<tr>
<td>3.06.04 Data Collection 3</td>
<td>3.40</td>
</tr>
<tr>
<td>3.06.05 Data Collection 4 – Interviews</td>
<td>3.42</td>
</tr>
<tr>
<td>3.06.06 Presentation of Results of Investigations</td>
<td>3.44</td>
</tr>
<tr>
<td>3.06.07 Summary of Research Design</td>
<td>3.45</td>
</tr>
</tbody>
</table>
## CHAPTER 4. SPECULATIVE DATA GENERATION

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.01</td>
<td>SUMMARY</td>
<td>4.1</td>
</tr>
<tr>
<td>4.02</td>
<td>INTRODUCTION</td>
<td>4.1</td>
</tr>
<tr>
<td>4.03</td>
<td>INVESTIGATION 1: HEART DEATHS VERSUS USE OF MOTOR VEHICLES</td>
<td>4.1</td>
</tr>
<tr>
<td>4.03.01</td>
<td>Introduction and Aims</td>
<td>4.4</td>
</tr>
<tr>
<td>4.03.02</td>
<td>Task</td>
<td>4.4</td>
</tr>
<tr>
<td>4.03.03</td>
<td>Participants and Method</td>
<td>4.5</td>
</tr>
<tr>
<td>4.03.04</td>
<td>Quantitative Results</td>
<td>4.8</td>
</tr>
<tr>
<td>4.03.05</td>
<td>Level 0 – Nonstatistical</td>
<td>4.10</td>
</tr>
<tr>
<td>4.03.06</td>
<td>Level 1 – Single Statistical Aspect</td>
<td>4.12</td>
</tr>
<tr>
<td>4.03.07</td>
<td>Level 2 – Inadequate Covariation</td>
<td>4.14</td>
</tr>
<tr>
<td>4.03.08</td>
<td>Level 3 – Appropriate Covariation</td>
<td>4.17</td>
</tr>
<tr>
<td>4.03.09</td>
<td>Discussion of Investigation 1</td>
<td>4.19</td>
</tr>
<tr>
<td>4.04</td>
<td>INVESTIGATION 2: HEIGHT VERSUS AGE</td>
<td>4.22</td>
</tr>
<tr>
<td>4.04.01</td>
<td>Introduction and Aims</td>
<td>4.22</td>
</tr>
<tr>
<td>4.04.02</td>
<td>Task</td>
<td>4.22</td>
</tr>
<tr>
<td>4.04.03</td>
<td>Participants and Method</td>
<td>4.23</td>
</tr>
<tr>
<td>4.04.04</td>
<td>Quantitative Results</td>
<td>4.27</td>
</tr>
<tr>
<td>4.04.05</td>
<td>Results – Q1&amp;2 – Height Versus Age</td>
<td>4.29</td>
</tr>
<tr>
<td>4.04.06</td>
<td>Results – Q3 – Height Versus Age By Sex</td>
<td>4.37</td>
</tr>
<tr>
<td>4.04.07</td>
<td>Discussion of Investigation 2</td>
<td>4.43</td>
</tr>
<tr>
<td>4.05</td>
<td>DISCUSSION OF SPECULATIVE DATA GENERATION AS EVIDENT IN INVESTIGATIONS 1 AND 2</td>
<td>4.45</td>
</tr>
<tr>
<td>4.06</td>
<td>INVESTIGATION 3A: HEIGHT VERSUS AGE (SURVEYS)</td>
<td>4.48</td>
</tr>
<tr>
<td>4.06.01</td>
<td>Introduction and Aims</td>
<td>4.48</td>
</tr>
<tr>
<td>4.06.02</td>
<td>Task</td>
<td>4.48</td>
</tr>
<tr>
<td>4.06.03</td>
<td>Participants and Method</td>
<td>4.48</td>
</tr>
<tr>
<td>4.06.04</td>
<td>Quantitative Results</td>
<td>4.49</td>
</tr>
<tr>
<td>4.06.05</td>
<td>Results – Q1&amp;2 – Height Versus Age</td>
<td>4.51</td>
</tr>
<tr>
<td>4.06.06</td>
<td>Results – Q3 – Height Versus Age By Sex</td>
<td>4.55</td>
</tr>
<tr>
<td>4.06.07</td>
<td>Discussion of Investigation 3A</td>
<td>4.57</td>
</tr>
</tbody>
</table>
4.11 INVESTIGATION 5B: HEART DEATHS VERSUS USE OF MOTOR VEHICLES (INTERVIEWS)

4.11.01 Introduction and Aims

4.11.02 Task

4.11.03 Participants and Method

4.11.04 Results – Overview

4.11.05 Results – Interview Dialogue

4.11.06 Discussion of Investigation 5B

4.12 GENERAL DISCUSSION – SPECULATIVE DATA GENERATION

CHAPTER 5. COORDINATE GRAPH PRODUCTION

5.01 SUMMARY

5.02 INTRODUCTION

5.03 INVESTIGATION 6A: TEMPERATURE CHANGE OVER TIME (SURVEYS)

5.03.01 Introduction and Aims

5.03.02 Task

5.03.03 Participants and Method

5.03.04 Quantitative Results

5.03.05 Level 0 – Nonstatistical

5.03.06 Level 1 – Single Statistical Aspect

5.03.07 Level 2 – Inadequate Covariation

5.03.08 Level 3 – Appropriate Covariation

5.03.09 Discussion of Investigation 6A

5.04 INVESTIGATION 6B: TEMPERATURE CHANGE OVER TIME (INTERVIEWS)

5.04.01 Introduction and Aims

5.04.02 Task

5.04.03 Participants and Method

5.04.04 Results – Overview

5.04.05 Initial Response Level 0 – Nonstatistical

5.04.06 Initial Response Level 1 – Single Statistical Aspect

5.04.07 Initial Response Level 2 – Inadequate Covariation

5.04.08 Initial Response Level 3 – Appropriate Covariation
5.04.09 Discussion of Investigation 6B ......................................................... 5.37

5.05 GENERAL DISCUSSION – COORDINATE GRAPH PRODUCTION... 5.38

CHAPTER 6. GRAPH INTERPRETATION ...................................................... 6.1

6.01 SUMMARY ......................................................................................... 6.1

6.02 INTRODUCTION .............................................................................. 6.2

6.03 INVESTIGATION 7: TELEPHONE RATE VERSUS CALL DURATION 6.5

6.03.01 Introduction and Aims ................................................................. 6.5

6.03.02 Task ............................................................................................ 6.5

6.03.03 Participants and Method .............................................................. 6.7

6.03.04 Results – Verbal Graph Interpretation (Q1) ............................... 6.7

6.03.05 Results – Numerical Graph Interpretation (Q2) ......................... 6.11

6.03.06 Results – Numerical Graph Interpretation (Q3 and Q4) .............. 6.13

6.03.07 Discussion of Investigation 7 ....................................................... 6.16

6.04 INVESTIGATION 8A: NOISE VERSUS NUMBER OF PEOPLE (SURVEYS) ................................................................. 6.18

6.04.01 Introduction and Aims ................................................................. 6.18

6.04.02 Task ............................................................................................ 6.18

6.04.03 Participants and Method .............................................................. 6.20

6.04.04 Results – Verbal Graph Interpretation (Q1 and Q4) ..................... 6.20

6.04.05 Results – Numerical Graph Interpretation (Q2 and Q3) .............. 6.23

6.04.06 Discussion of Investigation 8A ..................................................... 6.27

6.05 INVESTIGATION 8B: NOISE VERSUS NUMBER OF PEOPLE (INTERVIEWS) ................................................................. 6.29

6.05.01 Introduction and Aims ................................................................. 6.29

6.05.02 Participants and Method .............................................................. 6.29

6.05.03 Results ........................................................................................ 6.30

6.05.04 Discussion of Investigation 8B ..................................................... 6.37

6.06 GENERAL DISCUSSION – GRAPH INTERPRETATION ............... 6.38
LIST OF TABLES

Table 2.01. Selected Developments in the History of Graphing, Covariation and Functions ................................................................. 2.2
Table 2.02. Summary of Key Terms ................................................................. 2.50
Table 3.01. Numbers of Students Surveyed by Grade and Gender .................... 3.41
Table 3.02. Numbers of Students Interviewed by Gender and Grade .................. 3.44
Table 3.03. Summary of Research Design ..................................................... 3.45
Table 4.01. Characteristics of Four Levels of Speculative Data Generation .......... 4.3
Table 4.02. Response Counts and Percentages of Responses by Grade at each Level and Category of Speculative Data Generation .................. 4.9
Table 4.03. Response Counts and Percentages of Responses by Sex and Grade at each Level and Category of Speculative Data Generation for Q1&2 ................................................................. 4.28
Table 4.04. Response Counts and Percentages of Responses by Sex and Grade at each Level and Category of Speculative Data Generation for Q3 ................................................................. 4.28
Table 4.05. Response Counts and Percentages of Responses by Gender and Grade at each Level and Category of Speculative Data Generation for Task 3 Q1&2 ................................................................. 4.50
Table 4.06. Response Counts and Percentages of Responses by Gender and Grade at each Level and Category of Speculative Data Generation for Task 3 Q3 ................................................................. 4.51
Table 4.07. Summary Descriptions of Interview Dialogue for Q1&2 .................. 4.64
Table 4.08. Summary Descriptions of Interview Dialogue for Q3 ............. 4.65
Table 4.09. Response Counts and Percentages of Responses by Gender and Grade at each Level and Category of Speculative Data Generation for Task 4 ................................................................. 4.77
Table 4.10. Summary Descriptions of Interview Dialogue (Negative Covariation format) ................................................................. 4.99
Table 4.11. Summary Descriptions of Interview Dialogue (Positive Covariation format) ................................................................. 4.100
Table 4.12.  *Response Counts and Percentages of Responses by Gender and Grade at each Level and Category of Speculative Data Generation for Task 1 (revised)* ................................................. 4.111

Table 4.13.  *Summary Descriptions of Interview Dialogue* .......................... 4.119

Table 5.01.  *Response Counts and Percentages of Responses by Gender and Grade at each Level and Category of Coordinate Graph Production* ........................................................................... 5.9

Table 5.02.  *Summary Descriptions of Interview Dialogue* .......................... 5.24

Table 6.01.  *Characteristics of Four Levels of Verbal and Numerical Graph Interpretation* ................................................................................................................................. 6.4

Table 6.02.  *Response Counts and Percentages of Responses by Grade at each Level of Verbal Graph Interpretation to Q1* ................................................................. 6.8

Table 6.03.  *Response Counts and Percentages of Responses by Grade at each Level of Numerical Graph Interpretation to Q2* ......................................................... 6.11

Table 6.04.  *Response Counts and Percentages of Responses by Grade at each Level of Numerical Graph Interpretation to Q3 and Q4* ..................... 6.14

Table 6.05.  *Response Counts and Percentages of Responses by Gender and Grade at each Level of Verbal Graph Interpretation to Q1 and Q4* ....................... 6.21

Table 6.06.  *Response Counts and Percentages of Responses by Gender and Grade at each Level of Numerical Graph Interpretation to Q2 and Q3* .................. 6.24

Table 6.07.  *Students’ Level of Verbal Graph Interpretation by Level of Numerical Graph Interpretation* ........................................................................................................... 6.31

Table 7.01.  *Frequency of Student Responses at each Level of Speculative Data Generation for Task 1 by Level of Verbal Graph Interpretation and Numerical Graph Interpretation for Task 2* ........ 7.4

Table 7.02.  *Response Counts and Correlations Among Pairs of Response Levels* ............................................................................................................................................. 7.6

Table 7.03.  *Frequency of Student Responses at each Level of Speculative Data Generation for Tasks 3, 4 and 1 (revised) by Level of Speculative Data Generation for Task 3* ................................................... 7.8
Table 7.04.  Frequency of Student Responses at each Level of Speculative Data Generation for Task 1 (revised) by Level of Speculative Data Generation for Task 4 ................................................................. 7.9

Table 7.05.  Frequency of Comparison of Response Levels of Speculative Data Generation for Four Tasks ................................................................. 7.9

Table 7.06.  Response Counts and Correlations Among Pairs of Response Levels ........................................................................................................... 7.11

Table 7.07.  Frequency of Student Responses at each Response Level for Tasks 5 and 6 by Response Level for Tasks 4 and 5 ....................... 7.12

Table 7.08  Frequency of Student Responses at each Level of Numerical Graph Interpretation for Task 6 by Level of Verbal Graph Interpretation for Task 6 ........................................................................ 7.13

Table 7.09.  Frequency of Comparison of Response Levels of Speculative Data Generation, Coordinate Graph Production, Verbal Graph Interpretation and Numerical Graph Interpretation ...... 7.14
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.01</td>
<td>Forms of representing statistical covariation and skills to translate them.</td>
<td>1.4</td>
</tr>
<tr>
<td>2.01</td>
<td>Graph from Humphrey Bear, Nine Network, 2001.</td>
<td>2.10</td>
</tr>
<tr>
<td>2.02</td>
<td>Weather graph from WIN television news (19 January, 2002).</td>
<td>2.11</td>
</tr>
<tr>
<td>2.03</td>
<td>Percentiles of height versus age for females (Department of Health and Human Services, Tasmania, 2000).</td>
<td>2.12</td>
</tr>
<tr>
<td>2.04</td>
<td>Scatterplot multiple-choice test item.</td>
<td>2.21</td>
</tr>
<tr>
<td>3.01</td>
<td>Five possible response sequences.</td>
<td>3.10</td>
</tr>
<tr>
<td>3.02</td>
<td>A sample task and coding scheme from TIMSS (1995).</td>
<td>3.16</td>
</tr>
<tr>
<td>3.03</td>
<td>Task 1: Heart deaths versus use of motor vehicles.</td>
<td>3.22</td>
</tr>
<tr>
<td>3.04</td>
<td>Task 2: Telephone call rate versus call time, based on a newspaper extract.</td>
<td>3.23</td>
</tr>
<tr>
<td>3.05</td>
<td>Task 3: Height versus age.</td>
<td>3.24</td>
</tr>
<tr>
<td>3.06</td>
<td>Task 1 (revised): Heart deaths versus use of motor vehicles.</td>
<td>3.28</td>
</tr>
<tr>
<td>3.07</td>
<td>Task 4: Test scores versus study times.</td>
<td>3.30</td>
</tr>
<tr>
<td>3.08</td>
<td>Graph of average mathematics scores by students’ report on time spent daily on mathematics homework at grades 4, 8, and 12 from the National Assessment of Educational Progress, 2000 Mathematics Assessment.</td>
<td>3.31</td>
</tr>
<tr>
<td>3.09</td>
<td>Task 5: Temperature change over time.</td>
<td>3.33</td>
</tr>
<tr>
<td>3.10</td>
<td>Task 6: Noise level versus number of people.</td>
<td>3.34</td>
</tr>
<tr>
<td>4.01</td>
<td>Task 1 to assess Speculative Data Generation based on a newspaper extract (The Mercury, June 11, 1991, p.2).</td>
<td>4.5</td>
</tr>
<tr>
<td>4.02</td>
<td>Levels and categories of Speculative Data Generation for Task 1.</td>
<td>4.7</td>
</tr>
<tr>
<td>4.03</td>
<td>Student responses – Level 0 – Nonstatistical.</td>
<td>4.11</td>
</tr>
<tr>
<td>4.04</td>
<td>Student responses – Level 1 – Single Statistical Aspect.</td>
<td>4.13</td>
</tr>
<tr>
<td>4.05</td>
<td>Student responses – Level 2 – Inadequate Covariation.</td>
<td>4.16</td>
</tr>
<tr>
<td>4.06</td>
<td>Student responses – Level 3 – Appropriate Covariation.</td>
<td>4.18</td>
</tr>
<tr>
<td>4.07</td>
<td>Task 3 to assess Speculative Data Generation.</td>
<td>4.23</td>
</tr>
</tbody>
</table>
Figure 4.08. Coding scheme for levels and categories of response to Q1&2. 4.25
Figure 4.09. Coding scheme for levels and categories of response to Q3. 4.26
Figure 4.10. Student responses to Q1&2 – Level 1 – Single Statistical Aspect 4.30
Figure 4.11. Student responses to Q1&2 – Level 2 – Inadequate Covariation 4.32
Figure 4.12. Student responses to Q1&2 – Level 2 – Inadequate Covariation 4.34
Figure 4.13. Student responses to Q1&2 – Level 3 – Appropriate Covariation 4.36
Figure 4.14. Student responses to Q3 – Level 1 – Single Statistical Aspect 4.38
Figure 4.15. Student responses to Q3 – Level 2 – Inadequate Covariation 4.40
Figure 4.16. Student responses to Q3 – Level 3 – Appropriate Covariation 4.42
Figure 4.17. Student responses to Q1&2 – Level 1 – Single Statistical Aspect 4.52
Figure 4.18. Student responses to Q1&2 – Level 2 – Inadequate Covariation 4.53
Figure 4.19. Student responses to Q1&2 – Level 3 – Appropriate Covariation 4.55
Figure 4.20. Student response to Q3 – Level 1 – Single Statistical Aspect 4.56
Figure 4.21. Student response to Q3 – Level 2 – Inadequate Covariation 4.56
Figure 4.22. Student responses to Q3 – Level 3 – Appropriate Covariation 4.57
Figure 4.23. Prompts displayed to students to promote cognitive conflict for Q1 4.60
Figure 4.24. Prompts displayed to students to promote cognitive conflict for Q2 4.60
Figure 4.25. Prompts displayed to students to promote cognitive conflict for Q3 4.61
Figure 4.26. Response to Q2 with difficulty showing non-growth (G9f1) 4.68
Figure 4.27. Task 4 to assess Speculative Data Generation 4.73
Figure 4.28. Levels and categories of Speculative Data Generation for Task 4 4.75
Figure 4.29. Student responses – Level 0 – Nonstatistical 4.79
Figure 4.30. Student responses – Level 1 – Single Statistical Aspect 4.80
Figure 4.31. Student responses – Category 2A. Double and Group Comparison 4.81
Figure 4.32. Student responses – Level 2 – Inadequate Coordinate. .......................... 4.82
Figure 4.33. Student responses – Category 3A. Series Comparison. .......................... 4.84
Figure 4.34. Student responses – Category 3B. Table. ........................................... 4.86
Figure 4.35. Student responses – Category 3C. Coordinate Variable. ......................... 4.87
Figure 4.36. Student responses – Level 1 – Single Statistical Aspect. ......................... 4.88
Figure 4.37. Student responses – Level 2 – Inadequate Covariation. .......................... 4.89
Figure 4.38. Student responses – Level 3 – Appropriate Covariation. ......................... 4.91
Figure 4.39. Student responses to No Covariation task format (Q2). ......................... 4.92
Figure 4.40. Graphs used as prompts (Negative covariation). ................................. 4.95
Figure 4.41. Graphs used as prompts (Positive covariation). .................................. 4.96
Figure 4.42. Student response – Double Variable (G3f3). ....................................... 4.101
Figure 4.43. Student response – Double Comparison (G7m3). ............................... 4.103
Figure 4.44. Task 1 (revised), to assess Speculative Data Generation (The Mercury, June 11, 1991, p. 2). ................................................................. 4.109
Figure 4.45. Student responses – Level 0 – Nonstatistical. ...................................... 4.112
Figure 4.46. Student responses – Level 1 – Single Statistical Aspect. ......................... 4.112
Figure 4.47. Student responses – Level 2 – Inadequate Covariation. ......................... 4.113
Figure 4.48. Student responses – Level 3 – Appropriate Covariation. ......................... 4.114
Figure 4.49. Responses used as prompts to create cognitive conflict. ....................... 4.117
Figure 4.50. Survey response of an interviewee at Level 1. .................................... 4.120
Figure 4.51. Survey response of an interviewee at Level 2. .................................... 4.121
Figure 4.52. Survey responses of interviewees at Level 3. .................................... 4.122
Figure 5.01. Levels and categories of Coordinate Graph Production. ....................... 5.4
Figure 5.02. Task 5 to assess Coordinate Graph Production. .................................. 5.6
Figure 5.03. Student responses – Level 0 – Nonstatistical. ................................... 5.10
Figure 5.04. Student responses – Level 1 – Single Statistical Aspect. ....................... 5.12
Figure 5.05. Student responses – Level 2 – Inadequate Covariation. ....................... 5.14
Figure 5.06. Student responses – Level 3 – Appropriate Covariation. ....................... 5.16
Figure 5.07. Graphs used as prompt in interviews. ............................................. 5.22
Figure 5.08. Survey responses – Level 0 – Nonstatistical. ................................... 5.26
Figure 5.09. Survey responses – Level 1 – Single Statistical Aspect. ....................... 5.30
Figure 5.10. Survey responses – Level 2 – Inadequate Covariation. ....................... 5.33
Figure 5.11. Survey responses – Level 3 – Appropriate Covariation. ................. 5.36
Figure 6.01. Task 2 to assess Verbal Graph Interpretation (Q1) and Numerical Graph Interpretation (Q3 and Q4; Q2) based on a newspaper extract (The Mercury, 22 July, 1993, p. 17) .................. 6.6
Figure 6.02. Task 6 to assess Verbal Graph Interpretation (Q1 and Q4) and Numerical Graph Interpretation (Q2 and Q3) .................................................. 6.19
Figure 6.03. Student response – Level 2 – Inadequate Covariation. .................. 6.25
Figure 6.04. Student response – Level 3 – Appropriate Covariation. ................. 6.26
Figure 7.01. Proposed development order of four skills for understanding and representing statistical covariation. ................................................... 7.15
Figure 7.02. Responses from a Grade 3 female. ............................................. 7.16
Figure 7.03. Responses from a Grade 3 female. ............................................. 7.18
Figure 7.04. Responses from a Grade 5 male. .............................................. 7.19
Figure 7.05. Responses from a Grade 7 female. ............................................. 7.21
Figure 7.06. Responses from a Grade 7 male. .............................................. 7.22
Figure 7.07. Responses from a Grade 7 male. .............................................. 7.23
Figure 7.08. Responses from a Grade 9 male. .............................................. 7.24
Figure 8.01. Two graphs of the same data for X and Y. .................................. 8.18