OBSERVATIONS REGARDING PYRAMID NUMBERS.

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(Diagrams.)

The ancient structures of Egypt, especially the pyramids, have ever been regarded with the most profound interest. Travellers and historians find in them an everlasting theme for description. Geometricians also find in their designs, magnitude and dimensions, much matter for scientific speculations; and the mystic inspired by their age, grandeur, and mystery, is disposed to gather from their every feature some more or less fancifully conceived revelation or miracle.

Nor can we wonder at this. Egypt is the land of wonders. Great pyramids covering acres of land; colossi sitting silent in granite thrones; obelisks of prodigious height wonderfully carved from a single stone; and temples, sphinxes, and canals, of stupendous proportions. When we consider that all these monuments were hoary with age at the time of Herodotus, and that a close study of their works and hieroglyphics reveals that their builders had attained great knowledge in astronomy, geometry, architecture, engineering, and various arts, we may readily admit that our highest modern civilisation was cradled in the land of the Pharaoh's.

It is not my intention, however, to enter into the enquiry of Egyptian civilisation at present. The observations which I have to make are confined to the pyramid structures themselves. It is now well established that pyramidal structures were peculiarly characteristic of the most ancient civilisations of India, Babylon, Nineveh, Egypt, China, North America, Mexico, and even in islands of the Pacific; and that the whole or greater part of them are associated with sepultures for the dead. But while it is most probable that originally such monuments were built solely for commemoration and for the preservation of the remains of noble persons, there are also good reasons for supposing that some of them—such as the Great Pyramid of Gizeh or Cheops—fulfilled a double purpose. The Great Pyramid of Cheops covers a space of about 13·05 acres. If we make allowance for slight disturbance, due to pressure of the enormous superincumbent weight, we must assume that its designer intended its base to form a perfect square, and its shape a true pyramid. The various measurements of the most competent engineers only show a variation of 11, 13, and 19 inches in the length of each side, and with such doubtful data the side has been variously estimated at between 9,129 and 9,164 inches, and the mean of the five most careful measurements give a length of 9,137 inches, or 36,548 inches for the circuit of the four sides. Ferguson's, Dufell's, and Colonel Howard Vyse's measurements of height are the most reliable, and they only vary between $450\frac{3}{4}$ feet and 456 feet, or 5,472 English inches. Broadly speaking therefore its circuit represents about 100 inches for each day in the year, and its height almost exactly $\frac{3}{5}$lbs. of its side base. The orientation or eastward aspect is almost true, being 0 for South-East, + ·1 for North-East, + ·1 for South-West, and + ·0·636 for North-West. Subsequent settlement or earth tremors might easily account for these minute divergences from absolutely true orientation.

While rejecting the many fanciful interpretations of mystical writers drawn from known facts with respect to shape, dimensions, measurements, and orientation, I have long been convinced by the reasoning of sober minded investigators that the principal characteristics were probably determined as a base or fixed standard for measures of space and capacity; and if so, the shape and dimensions themselves might have been suggested to the skilled geometricians of the time by reference to some astronomical fact of importance known to them, in conjunction with significant properties of number and proportion discovered by them to belong to the structure of cubes in pyramidal form. That men who taught the modern world mensuration and astronomy, should strive to attain a sure method for securing uniformity in standards as applied to weights and measurements, is a most reasonable supposition. That these standards should be symbolised by some striking or well-known astronomical fact, is in the highest degree probable, and corresponds exactly with the idea of the French astronomers, who determined the length of their metre in relation to the ascertained length of a meridian line drawn from the Pole to the Equator. (The metre representing the tenth millionth, or 39·37079 English inches; the centimetre being one hundredth of a metre. The gramme or standard of weight is derived from the centimetre, *i.e.*, a cubic centimetre of distilled water at the temperature of maximum density, nearly equal to ·0022054 of an English avoirdupois pound, or 15,438 English grains.)

Impressed with this idea, and with the conviction that the Egyptian builders were adepts in the construction of models, I sought to obtain some light upon these matters by studying the numerical combinations of simple cubes built upon the pyramid type. I was guided to a considerable extent in these investigations by the wide prevalence of multiples of 7, 12, and 10, in the existing subdivisions of time, space, weight and value. How has it come about, for example, that a certain sacredness attaches to the number 7? Why was the important division of the year (a week) determined to be seven days, for it was in common use long before the birth of Moses? Why was the seventh day originally set apart as the Sabbath? Why have we the day divided into two parts of 12 hours each, and why do multiples of 12 so commonly appear in weights and measures, especially in astronomical divisions?

In many combinations conducted with the hope of throwing light on such matters, I failed to get any remarkable indications, with three important exceptions. These three exceptions possess so many remarkable proportions and numbers relating to existing sub-divisions of weights, measures, and values, and especially with the proportions and dimensions of the Great Pyramid, that I have been induced to risk the appellation of "pyramid mystic," and to lay the remarkable results before the members of this Society.

The models which appear before you have each some particular claim to notice, and whether any of them may offer sufficiently remarkable characteristics or not as bearing upon the Great Pyramid, they are all well worthy of close attention as offering a natural solution to the genesis of particular numbers as used in sub-divisions or measures of time, space, weight, or value.

PYRAMID OF ODD NUMBERS, HAVING 7 AS A BASE.

As shown in diagram, the most remarkable characteristic is the fact that the cube root of its basal layer, 49 or 7^2, enters into and agrees with all the important dimensions of the Great Pyramid, including length of complete circuit; length of side; height; length of Egytian cubit; English inch; and through the latter it harmonises in the most obvious and simple multiples with these dimensions and the days in the year, days in the lunar month. Other natural proportions of the three angled sides of pyramid connote the months in the quarter and year; while its aggregate number of cubes, 84 or 7 times 12, suggest the alliance of 7 and 12 in measurement of time.

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Demonstration indicating that the Great Pyramid dimensions were probably determined by the radix of sacred number (7), which in itself has probably been selected because the cube root of its square contains nearly the exact figures representing the known days in the year:—

$$\text{Radix } (\sqrt[3]{7^{\cdot 2}}) = \text{R} = 3{\cdot}6593.$$

1. Circuit of pyramid in inches 36593 = 10000R
2. Length of each side (4) ,, ,, 9148 = 10000R
3. Height of pyramid ,, ,, $5488^{\cdot 9} = \dfrac{^3(10000\text{R})}{2}$ $= 457^{\cdot 9}$ feet

NOTE.— $\frac{3}{2}$ Height proportion may have been selected because 3 expresses the number of dimensions in a cube: and 1 ÷ 2 exactly expresses the relative elevation surface of a triangle and square resting upon a common base and of equal vertical height at its maximum, in a vertical line drawn at right angles to base line.

4. Principal unit of measurement in inches $25^{\cdot 41} = \frac{1000R}{12^{\cdot 2} \text{ or } 144}$
Nearly equal to existing cubit in Egypt.

NOTE— $12^{\cdot 2}$ most probably was adopted as a divisor, because curiously enough the actual number of square cubes contained in a pyramid of even numbers, which most nearly approaches the number of days in a year, is 364, and the base of such pyramid or 1st layer contains 12 × 12 cubes or 144: the second layer in importance succeeds it with 10 × 10 cubes or 100 (see plan).

5. Cubits in circuit of pyramid, No. 1440 $= \frac{1000R}{12^2}$

$$= \frac{10000R}{10000R \div 12^2} \text{ or } \frac{10000R}{25^{\cdot 41}}$$

6. Ditto in each side (mean) 360. $\left(\frac{10000R}{10000R \div 12^{\cdot 2}}\right) \div 4$

7. Unit or year or 1 $= \frac{10000R}{10000R}$ or $\frac{\sqrt[3]{7^{\cdot 2}}}{\sqrt[3]{7^{\cdot 2}}}$

8. Days in Week or 7 $= \frac{7^{\cdot 2}}{7}$ or $\frac{49}{7}$

9. Months in Year or 12 = Angles on 4 faces 4 × 3 : also equal to base of a simple pyramid of even numbers whose aggregate represents 364: also, the seventh of the aggregate of a simple pyramid of odd having 7 for its base.

10. Lunar Months in year or $13^{\cdot 3}$ nearly $(13^{\cdot 07}) = \frac{10000R}{7 \times 4}$

SQUARE PYRAMID OF MIXED ODD AND EVEN NUMBERS, HAVING FOR A BASE $(7 \times 2)^2$ OR $14^2 = 196$.

Perhaps this forms the most interesting of all the combinations. Its natural proportions and naturally related numbers are most suggestive.

The following combinations are most striking :—

1. If we take either the exposed cubes on the margin of each layer, or the total faces of distinct cubes in the four sides, the aggregate comes exactly to 365, or the exact number of days in the year; and therefore the proportional number of cubes on each triangular face is $91\frac{1}{4}$, corresponding to days in the quarter of a year.
2. If we now take the basal layer alone, we find the exposed number of cubes in the square to be 52, corresponding to weeks in the year.
3. If again we take the aggregate of all cubes in the pyramid, we find they amount to 1,015, and if this number be multiplied by 36, or 4 times 9 (the latter number representing the number of verticle angles on faces of the four wedges or prisms of which the pyramid is built, as indicated by its diagonals), we obtain 36,540, or within 8 inches of the best actual measurements of its present state, which has no doubt undergone some slight settlement due to superincumbent pressure.
4. A quarter of this gives 9,135 inches, or within 2 inches of the mean of the best actual measurements obtained by competent investigators.
5. If we now take the square of its basal layer, 14 × 14, we get 196, and it is remarkable that if this number be multiplied successively by half the side, and by the number of sides, *i.e.*, 196 × 7 × 4, we get 5,488, or within 16 inches of the best estimates of the present height of the Great Pyramid, any two of which differ far more seriously with each other than this curious combination.

6. The basal layer has 13 distinct cubes in each side, corresponding to number of weeks in each quarter, which the side typifies naturally; while the three angles of each triangular face makes 12, corresponding with the months in the year or hours in the day.

These combinations are all natural to the particular structure, and are not selected in arbitrary or forced way as in many suggestions found in works referring to the pyramids.

Square Pyramid of Even Numbers Having 12 for its Base.

The remarkable characteristic of this pyramid is that—

1. The aggregate of all the cubes, if capped with an odd one as a finishing point, numbers 365, corresponding to the number of days in the year. It has 12 cubes along the basal layer of each side, corresponding to months. There are exactly 36 cubes in each triangular face, and 144 in basal layer. If each of these be multiplied by the number of cubes in side of 2nd layer, and taken as divisor of the circuit and side of pyramid they give results which almost exactly correspond with the existing cubit of Egypt.

The same result is very closely attained bp multiplying the aggregate number of cubes (365) by 7, and dividing the result by the square of the second layer (100).

2. But perhaps the more interesting numbers in this pyramid of even numbers are those of the cubes of the exposed sides of squares, and the aggregates of the cubes in each layer.

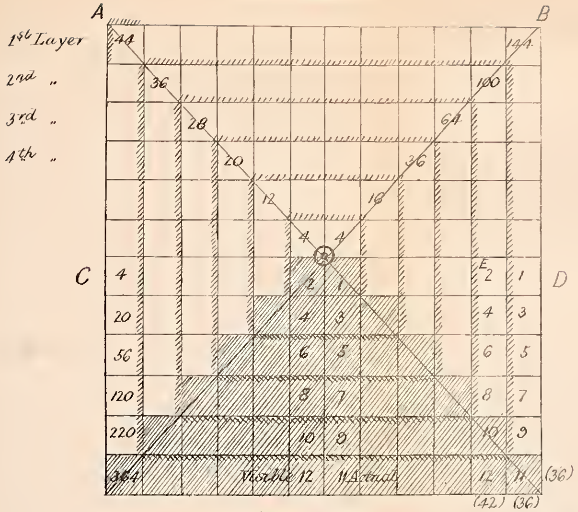
It is singular that in the first series the sequence 1, 4, 12, 20, should exactly correspond with the sequence of English standards of money value, viz.: Farthing, farthings in penny, pennies in a shilling, and shillings in a pound.

The figures of the base, 12 and 144, are associated with sub-divisions of square measured multiples or sub-divisions of 28, as 7, 14, 28, 56, 112, 2,240 as in sub-divisions of weight; and in the second series of aggregates we have in the second layer the numbers 10, 220, and in the basal exposed margin of circuit 44, all suggestive of some connection with reasons which originally entered into the determination of subdivision of 44, 220, 440, 1,760, in the English mile.

Conclusion.

Taken by themselves the remarkable coincidences with known facts relating to measurement of time and space might only be construed as simple examples of the facility with which many numbers may be made to coincide with known

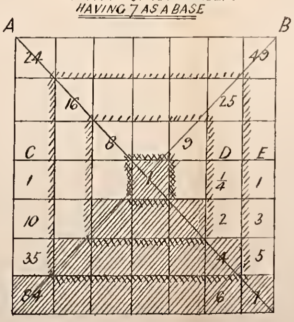
PYRAMID OF EVEN NUMBERS HAVING 12 AS A BASE



EXPLANATION OF INDICES

	LAYERS OF CUBES 1st	2nd	3rd	4th	5th	6th	
B Cubes in no:	144	100	64	36	16	4	= 3[illegible]
C Cubes in and above no:	364	220	120	56	20	4	= 364
A Actual Cubes on outer marginal layer visible from above	44	36	28	20	12	4	= 144
E Cubes visible in elevation of each face	12	10	8	6	4	2	= 42
D Actual proportion of distinct cubes forming each face	11	9	7	5.	3	1	= 36
Ditto for four faces	44	36	28	20	12	4	= 144

PYRAMID OF ODD NUMBERS HAVING 7 AS A BASE



EXPLANATION OF INDICES

	LAYERS OF CUBES 1st	2nd	3rd	4th	All
A Actual Cubes on outer marginal layer visible from above	24	16	8	1	= 49 or (7 x 7)
B Cubes in	49	25	9	1	= 84 or (7 x 12)
C Cubes in and above	84	35	10	1	= 84 or (7 x 12)
D Proportion of distinct cubes forming each face	7	5	3	1	= 16
E Cubes visible in each face	6	4	2	$\frac{1}{4}$	= $12\frac{1}{4}$ or ($\frac{7 \times 7}{4}$)

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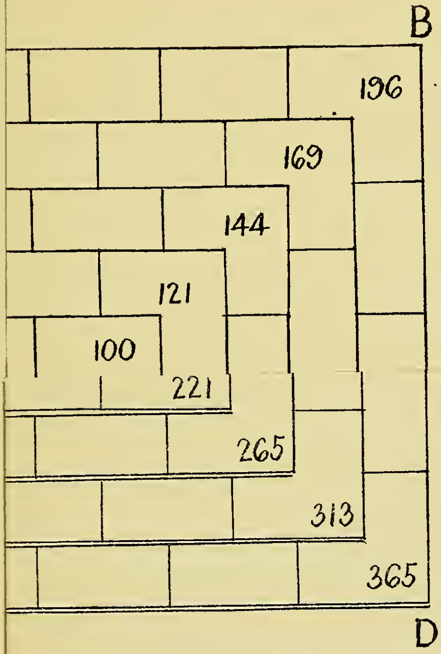
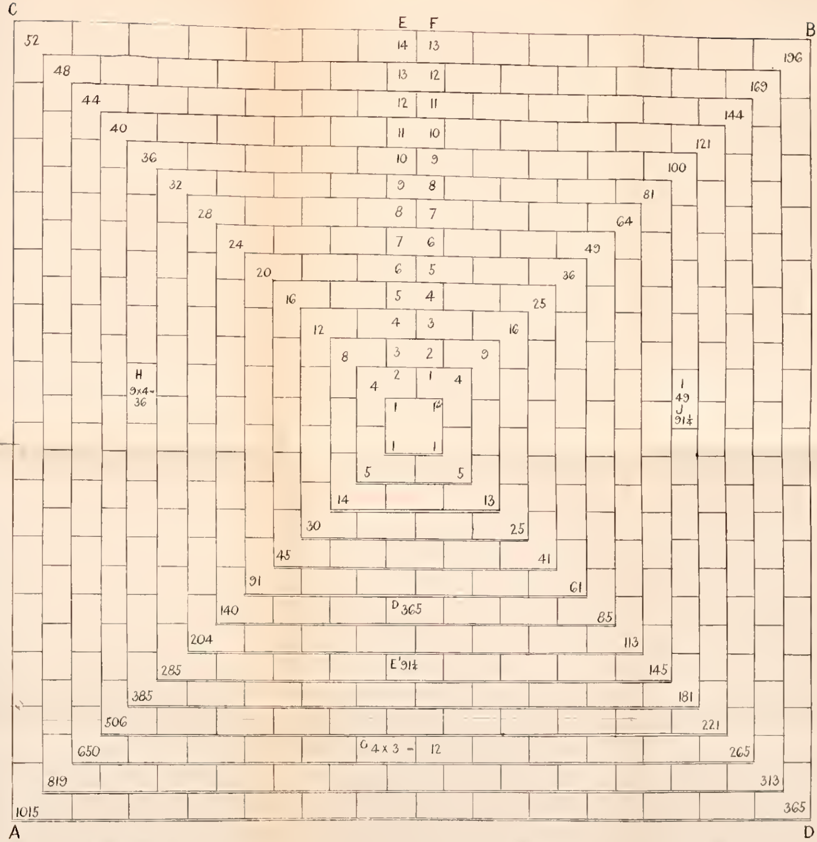


cubes on and above each layer ; [a] Basal = 1015
" on each layer ; [a] " = 196
xposed on the four vertical faces of each layer ; [a] " = 52
" " " " of all layers ; " = 365
bes seen on each of the four faces of each layer : " = 13
" " " " " " " " all layers : Aggregate = 91¼
" " " " " " " " each layer : Basal = 14
ngles on four faces 4 x 3 = 12
rtical faces of the four triangular based
ms bisected by the diagonals of square. 4 x 9 = 36
ing base of each of the four wedges or prisms 7 x 7 = 49
cubes on each " " " " " " = 91¼
yramid

SQUARE PYRAMID OF ODD AND EVEN NUMBERS OF CUBES

HAVING FOR ITS BASE $(7 \times 2)^2$



AH = 1015 × 36 = 36540. or circumference of Pyramid in inches

$\frac{AH}{4}$ = 9135 or length of one side ditto

$\frac{B(F)K}{(2)}$ = 196 × $\frac{14}{2}$ × 4 3488 or height of Pyramid ditto

$\frac{AH}{4}$ ÷ 365 25·03 inches nearly the length of existing cubit of Egypt

C^a = 52 weeks of seven days in year

D^a = 365 days in year

E^a = 13 weeks in each quarter

$\frac{D^a}{C^a}$ = 7 days in week

G = 12 months in year

A Represents the aggregate number of cubes on and above each layer : a Basal = 1015

B " " " " on each layer : a " = 196

C " " " " " exposed on the four vertical faces of each layer : a " = 52

D " " " " " " " of all layers : " = 365

E " " actual number of cubes seen on each of the four faces of each layer : " = 13

E' " " " actual " " " " " " all layers : Aggregate 91¼

F " apparent " " " " " each layer Basal = 14

G number of distinct angles on four faces 4 × 3 = 12

H " " " of angles on vertical faces of the four triangular based wedges or prisms bisected by the diagonals of square 4 × 9 = 36

I " " " of cubes forming base of each of the four wedges or prisms 7 × 7 = 49

J " aggregate number of cubes on each " " " " " 91¼

K " number of sides of pyramid

measurements or proportionals relating to the earth's diameter, circumference, distance from the sun, annual period of revolution, etc.; for it is easy by slight variations of any root, arbitrarily made, and arbitrarily selected multiples, to make any number approximate to some important terrestrial measurement, provided that the computer is himself previously aware of the proportional, size, or measurement, with which a show of correspondence is desired. Much of the so-called remarkable coincidences of mystical writers are of this class; for it not unfrequently happens that the same root measurement, by slight alteration, is worked up to bring about coincidences with very different things. Thus Mr. Piazzi Smith, by taking the height of the niche of the Queen's Chamber of the Great Pyramid as 182·62, and multiplying it by 2, he obtains 365 242, equivalent to the days in the year; and again by arbitrarily taking the same dimensions as 185, and multiplying it successively by 3·1416 and 10, he obtains 5,812, which he arbitrarily concluded to be the height of the Great Pyramid in inches. But curiously enough the same dimensions, 182·62, multiplied by 10 and divided by 2 (why not at once multiply by $5^{\cdot 2}$) is made to show an approximate to length of one of the sides in inches. These are common examples of the facility with which many fancy the discovery of purposeful design in numbers or dimensions, when dealt with in a fanciful and arbitrary way.

It seems to have been forgotten by such persons that any root figure, by the *arbitrary selection* of a multiplier or divisor, may be made to coincide exactly with any other number *provided the manipulator knows beforehand the number or proportional* with which correspondence is sought to be established.

But making all allowance for the vagaries of the mystics, there are many legitimate subjects of enquiry, upon which some light might be thrown by the careful investigation of ancient structures At the present day it is remarkable how largely the numbers 7, 12, and 10, or simple multiples of these enter into standards of space, time, weight, and value. It is easy to imagine how 10 was seized upon so frequently as a standard of measurement; for counting by means of the digits of the two hands so universal and so natural at once suggests a probable reason; but the reasons for the original selection of 7 and 12 for a similar purpose are not so easily conceived.

What, for example, were the determining causes for the selection of the many sub-divisions of weights, values, time, lineal and square measure?

Why have we a sequence of 4, 12, 20, in English money in sub-divisions of the penny, shilling and pound; of 14, 28, 56, 112, 2,240, in sub-divisions of a ton weight; of 44, 440, 1,760, in sub-divisions of the English mile; of multiples of 12 in square measure; of either 8 or 7 as a root of wine measure?

8 × 1	gall.:	8 × 42 7 × 48	tierce:	8 × 62 7 × 72	hogshead
8 × 84 7 × 96	puncheon:	8 × 126 7 × 144	pipe:	8 × 252 7 × 288	tun

Then going to the survivals of ancient systems of linear measurement, how can we account for the origin of lineal measures, such as—

The English foot ...	Equivalent to	12	English inches
The ancient "Pied de Roi" of France ...	"	12·79	"
The German elle	"	22·58 22·9	" "
The Italian pié ...	"	22·428	"
The common guerze of Persia	"	25	"
The pic of Turkey ...	"	26·8	"
The braccio of Ancona	"	25·33	"
The short picha of Greece	"	25	"
The long " "	"	27	"
The existing derah or cubit of Egypt ...	"	25·488	"
Jewish cubits ...	"	25·61 24·74	"

May it not be possible therefore that the ancient draftsmen or modellers of pyramids had seized upon many of these characteristics shown in the forms and figures referred to, both for sub-divisions of measures and weights, and also to typify in their important fixed standards some of the more remarkable facts of astronomy then known to them?