Using Fathom® statistical education software in high school to examine students’ acceptance of virtual simulation and use of simulation to model sample size when sampling from large and infinite populations

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Submitted in fulfilment of the requirement for the degree of PhD,
University of Tasmania
November, 2012

Declarations, Statement of Co-authorship, Abstract, Acknowledgements, Table of Contents, List of Tables, List of Figures,
Chapters 1 – 5,
References, Acronyms & Initialisations, Glossary, and Mathematical Symbols
Declarations

Declaration of Originality
This thesis contains no material which has been accepted for a degree or diploma by the University or other institution, except by way of background information and duly acknowledged in the thesis, and to the best of my knowledge and belief no material previously published or written by another person except where due acknowledgement is made in the text of the thesis, nor does the thesis contain any material that infringes copyright.

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The research associated with thesis was conducted in accordance with the Human Research Ethics Committee (Tasmania) Network (HERCS) (Approval No. H009790), the Department of Education Tasmania (Ref. 672670), and the Teachers Registration Board of Tasmania Code of Professional Ethics for the Teaching Profession in Tasmania. To the researcher’s and the supervisors’ knowledge no concerns regarding the ethical nature of the study or the personal conduct of the researcher were raised by the schools, the principals, the colleague teachers, parents or the students, HERCS or the Department of Education Tasmania.

Funding
This project was jointly funded by the Australian Research Council (ARC) through an Australian Postgraduate Award Industry (APAI) Linkage project grant (LP0669106) and by the publisher of Fathom software Key Curriculum Press of Emeryville, CA.

Signed, 16\textsuperscript{th} November, 2012

Anthony Frederick Bill
Statement of co-authorship

The following people contributed to the publication of the work undertaken as part of this thesis:


Candidate/author 1 (80%), author 2 (10%), and author 3 (10%)

Author 2 and author 3 co-contributed to the conduct of the study and reviewed the paper.

We the undersigned agree with the above stated "proportion of work undertaken" for each of the above published (or submitted) peer-reviewed manuscripts contributing to this thesis:

Signed and dated: Anthony Frederick Bill 27/2/12
Sally Henderson 27/2/12
John Penman 27/2/2012
Abstract

Statistical literacy is regarded as essential for good citizenship, employment, and practical day-to-day living. The ubiquitous nature of data and computers in contemporary society has increased both the need for statistical literacy and the means of developing statistical literacy.

This study investigated students’ acceptance of Fathom® virtual simulation and resampling as a legitimate mathematics tool, the teaching and learning of the explicit determination of sample size when sampling from large populations, and students’ development of use of Fathom statistics education software.

The study was conducted as a three-week long classroom unit of work taught in two Year 9 classes and a detailed study of twelve students in Tasmania, Australia. Pedagogical best practice principles derived from statistics education research guided the study. These included engagement with the big ideas of statistics, active learning and data sets students can understand and value, statistical enquiry that cultivates statistical habits of mind, the use of technology tools that allows students to explore data and concepts, mathematical experiences of substance, provision of a developmental pathway for students to study statistics at more senior years, and authentic assessment.

Fathom was developed for senior high school and tertiary study, and its use in Australian high schools is relatively novel. Students’ unfamiliarity with the software presented at least two challenges: developing acceptance of Fathom’s virtual resampling probability simulator as a legitimate mathematical tool and acquiring basic fluency in the software’s use such that the software was not a constraint on learning. Students’ acceptance of the probability simulator was cultivated purposefully through a process of formal statistical enquiry where students examined the fairness of the Fathom virtual die. Students’ development of use of Fathom re-sampling was examined from the three aspects of key terminology, graphical data representations, and their relationship with Fathom. The principles of instrumental genesis guided the introduction to, and the examination of, students’ use of Fathom.

Sample size is presently ignored in the high school curriculum, and students may complete formal school education with unsophisticated notions of sample size, possibly first acquired in upper primary school. The sample size model \( e = \pm 1/\sqrt{n} \), which
relates the sample size $n$, to the margin of error $e$, of the accuracy of measurement, was used in this study. A foremost consideration was that the model was potentially accessible and that students could apply their understanding in a real-life context. Large populations were studied because formal mathematical treatment is relatively simple. Students’ work samples were assessed using the SOLO taxonomy, and situated abstraction was used to observe students’ development of understanding of selected mathematical concepts.

The study concluded that a process of statistical enquiry may be used both to promote acceptance of virtual simulation and to foster the development of statistical “habits of mind.” The sample size model $e = \pm 1/\sqrt{n}$ has application in Year 9 principally to mathematise traditional Law of Large Numbers activities, where the computing power of virtual simulation allows exploration of very large sample sizes. The introduction of re-sampling and the sample size model in Year 9 provides the foundation for the consideration of contextual tasks in more senior school years. The study suggests that Fathom is suitable for Year 9 students, but recommends further research in the use of re-sampling to exploit fully the software’s potential.
Acknowledgements

Many people have been influential during the research and writing of this thesis. I would like to thank the following people for their encouragement and support during this interesting, and occasionally, challenging time.

My partner Louise Oxley, without whose love, loyalty, and steadfast support this thesis would not have been completed. Thank you, my dear friend.

My daughter Emily, who, in the time of this thesis, has grown into a highly talented, intelligent, beautiful, gracious woman of wisdom and maturity.

My co-supervisor Emerita Professor Jane Watson, whose professionalism, dedication, and untiring support throughout the writing of this thesis were astonishing. Thank you Jane.

My co-supervisor Associate Professor Rosemary Callingham, whose sensible strategic advice has been of immense value.

My adviser, Mr. William Finzer, of KCP Technologies, Emeryville CA, who helped shift my gaze to the international community.

My two colleague teachers whose generosity, professionalism, and kindness are gratefully acknowledged.

I would also like to acknowledge and thank:

The students who participated in the study

Colleagues and staff at the University of Tasmania

My colleagues in the wider statistics education research community

The three industry partners The Australian Bureau of Statistics; Key Curriculum Press, USA; and the Noel Baker Centre for School Mathematics, Prince Alfred College, South Australia.
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