The Magnetic Rayleigh-Taylor Instability

by

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I declare that this thesis contains no material which has been accepted for a degree or diploma by the University or any other institution, except by way of background information and duly acknowledged in the thesis, and to the best of my knowledge and belief no material previously published or written by another person except where due acknowledgement is made in the text of the thesis, nor does the thesis contain any material that infringes copyright.

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The following people contributed to the publication of work undertaken as part of this thesis.

The magnetic Rayleigh-Taylor instability for inviscid and viscous fluids [11]
Kain Chambers (80%), Larry Forbes (20%)

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We the undersigned agree with the above stated proportion of work undertaken for each of the above published (or submitted) peer-reviewed manuscripts contributing to this thesis.

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Abstract

The Rayleigh-Taylor instability (RTI) arises whenever two fluids with different densities are arranged such that the heavier fluid sits above the lighter fluid, with a sharp interface in between. The magnetic Rayleigh-Taylor instability (MRTI) has the further complication due to the presence of a magnetic field throughout both media. The two fluids in question may also have differing magnetic properties, such as the magnetic permeability. When the fluids in consideration are in fact plasmas comprised of charged particles, induced currents, magnetic fields and Lorentz forces can all act in ways that will affect the stability of the system.

The RTI has widespread applications in atmospheric physics, oceanography, meteorology, laboratory plasma physics, nuclear reactors, inertial confinement fusion as well as the field of astrophysics, where the instability plays an important role in supernova explosions, accretion discs, plasma jets and H II regions (clouds of gas in which star formation has recently taken place) amongst others. It is closely related to two other hydrodynamic instabilities, namely the Kelvin-Helmholtz instability (KHI) and the Richtmyer-Meshkov instability (RMI).

This thesis considers in detail several different flow configurations in which the RTI arises. A key feature of these configurations is that small wavelike disturbances to the flow are unstable. These configurations are studied to determine the behaviour of these unstable flows. A particular focus is the effects due to the presence of a magnetic field, and the mechanisms that alter the flow in the magnetic case.

The thesis begins by considering two dimensional planar flow in Cartesian geometry in which the amplitude of the waves is small compared with their wavelength. The flow is assumed to be incompressible and inviscid. In this scenario, linear theory is used to derive a closed form solution for the evolution of the interface between the two fluids. This solution shows quantitatively how the position of the interface depends on the ratio of the densities of the two

fluids, the wavelength of the disturbance, as well as the strength and direction of the applied magnetic field.

The unstable nature of the RTI means that after some finite time, the amplitude of the waves will grow to a size comparable with their wavelength, and in this scenario, linear theory is not appropriate. For this reason, a non-linear model is considered, again for two dimensional planar flow in Cartesian geometry. The flow in this case is considered to be weakly compressible, and viscous. Results in the non-linear case are obtained by use of a combination of streamfunction, spectral and finite difference techniques. The results show qualitatively various non-linear phenomena such as interface roll-up, fingering and bubble formation. It is shown in particular how different initial conditions give rise to outcomes that are very different in terms of the geometry of the interface between the two fluids, primarily the differences between a single mode disturbance and a multi mode disturbance to the interface at time $t = 0$.

The final problem studied in this thesis considers two dimensional flow in circularly symmetric cylindrical geometry. The configuration in this case is comprised of a heavy fluid surrounding a light fluid, and gravity is directed radially inwards. A massive object is located at the centre of the light fluid, and it behaves like a line dipole both for fluid flow and magnetic field strength. In the non-linear, weakly compressible, viscous regime, the initially circular interface between the two conducting fluids evolves into plumes, dependent on the magnetic and fluid dipole strengths and the nature of the initial disturbance to the interface. A spectral method is presented to solve the time-dependent interface shapes, and results are presented and discussed. Bipolar solutions are possible, and these are of particular relevance to astrophysics. The solutions obtained resemble structures of some HII regions and nebulae.

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