Shallow Water Catamaran Wash –
Simple Characterisations for a
Complex Phenomenon

Alexander Robbins
BEng MPhil C.Eng MRINA

Submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

Australian Maritime College
University of Tasmania
March 2013
Photo 1 – “Dangerous waves caused by ships - Enter pier at own risk”
Declarations

Statement of Originality
This thesis contains no material which has been accepted for a degree or diploma by the University or any other institution, except by way of background information and duly acknowledged in the thesis, and to the best of my knowledge and belief no material previously published or written by another person except where due acknowledgement is made in the text of the thesis, nor does the thesis contain any material that infringes copyright.

Authority of Access
This Thesis may be made available for loan and limited copying in accordance with the Copyright Act of 1968

Copyright
I warrant that I have obtained, where necessary, permission from the copyright owners to use any third-party copyright material reproduced in the thesis (e.g. questionnaires, artwork, unpublished letters), or to use any of my own published work (e.g. journal articles) in which the copyright is held by another party (e.g. publisher, co-author).


Alex Robbins
March 2013
Statement of Contribution

Chapters 3 to 7 have been prepared as scientific manuscripts (papers), as identified on the title page for each chapter. In all cases conceptualisation, research programme design, experimental design, physical testing, numerical analysis, interpretation, and manuscript preparation were the primary responsibility of the candidate. Accordingly he was first-named author on all papers. However, they were carried out in collaboration with supervisors as co-author's, as outlined below.

Paper 1 - The Decay of Catamaran Wave Wake in Shallow Water.
Paper 2 - Vessel Trans-critical Wave Wake, Divergent Wave Angle and Decay.
Paper 3 - Subcritical Wave Wake Unsteadiness.

For these papers, Giles Thomas provided lead consultancy advice and manuscript preparation assistance; Gregor Macfarlane provided testing facility support, consultancy advice and manuscript preparation assistance; Martin Renilson and Ian Dand provided top level consultancy advice and manuscript preparation assistance.

[Candidate: 85%, Giles Thomas, Gregor Macfarlane, Martin Renilson and Ian Dand; 15%]

Paper 4 - Vessel Wave Wake Characterisation using Wavelet Analysis.

For paper four Walid Amin provided numerical calculation assistance and consultancy advice; Giles Thomas provided lead consultancy advice and manuscript preparation assistance; Gregor Macfarlane, Martin Renilson and Ian Dand provided top level consultancy advice and manuscript preparation assistance.

[Candidate; 85%, Walid Amin; 10%, Giles Thomas, Gregor Macfarlane, Martin Renilson and Ian Dand; 5%]

Paper 5 – When is Water Shallow?

For paper five Giles Thomas, Gregor Macfarlane, Martin Renilson, and Ian Dand provided top level consultancy advice and manuscript preparation assistance.

[Candidate: 95%, Giles Thomas, Gregor Macfarlane, Martin Renilson and Ian Dand: 5%]
Co-author Agreement.

We the undersigned agree with the above stated “proportion of work undertaken” for each of the above published (or submitted) peer-reviewed manuscripts contributing to this thesis:

------------------------------
Associate Professor Giles Thomas
Australian Maritime College,
National Centre Maritime Engineering and Hydrodynamics,
University of Tasmania.

------------------------------
Dr. Gregor Macfarlane
Manager - Towing Tank and Model Test Basin,
Australian Maritime College,
National Centre Maritime Engineering and Hydrodynamics,
University of Tasmania.

------------------------------
Professor Martin Renilson
Dean, Maritime, Higher Colleges of Technology,
United Arab Emirates and
Adjunct Professor,
Australian Maritime College,
National Centre Maritime Engineering and Hydrodynamics,
University of Tasmania.
Dr. Ian Dand  
Senior Consultant Hydrodynamicist,  
BMT Isis,  
Fareham, United Kingdom

Dr. Walid Amin  
Post-Doctoral Research Fellow,  
Australian Maritime College,  
National Centre Maritime Engineering and Hydrodynamics,  
University of Tasmania.
Abstract

This thesis explores catamaran generated wash in shallow water, with the specific goal of simplifying this highly complex phenomenon into user friendly characterisations.

The wash caused by marine traffic in coastal or inland waterways can have significant effects on waterway users and shore goers within the littoral zone. Typically wash is a minor irritant affecting only the enjoyment of others. However, given certain conditions, wash can also be a significant safety hazard, with one death directly attributed to it.

Vessel wash can also have a major impact on the environment with wash related environmental degradation being reported in many countries, particularly those with high speed marine traffic. The potential exists that significant and irreversible changes will occur to the environment because of it. Accordingly wash environmental impact is no longer seen as of secondary importance to successful operations, but in some cases has become the key requirement.

Successful mitigation of this wash hazard clearly involves the source of the wash, being vessels and their operation. In response to this hazard, authorities have implemented mitigation strategies, typically route planning and speed restrictions, with limited success. Fundamentally, regulation without understanding of the wash phenomena cannot be truly effective. Accordingly a series of simple wash characterisations are proposed to better define shallow water wash. These characterisations can be utilised by naval architects, operators and regulators in their assessment of wash and in turn effect hazard reduction.

A “thesis by publication” approach has been taken, comprising five published papers. As is the case for a conventional thesis, the outcome is of a sustained and cohesive theme throughout, (being reflected in the thesis’ title). The first four papers each examine a separate wash characterisation, outlining performance in shallow water. These papers were the outcome of physical tests held in the Australian Maritime College model test basin, Tasmania.

The first and second papers, on wash decay and divergent wave angle respectively, both confirmed that these parameters vary significantly with water depth, (as well as displacement and hull form), and can be utilised as shallow water characterisations. The third paper investigated unsteadiness that was recorded during experiments and it was concluded that
this is due to soliton generation. Furthermore, such highly non-linear waves are known to be a significant safety risk.

The fourth paper provides proof of concept that wavelet analysis is successful in characterising vessel wash. The wavelet methodology allows differentiation of various hull forms from their wave patterns alone. The fifth and final paper is a summary of the thesis findings, placing them within the global context of existing vessel performance indicators such as resistance, propulsion and manoeuvring. In turn this paper establishes that the simple wash characterisations proposed are a significant extension of existing wash knowledge and also incorporates a new and novel method of wavelet analysis.

Accordingly the thesis achieves its goals of clearly defining shallow water wash and establishing a series of simple wash characterisations
Acknowledgements

The author sincerely acknowledges the following people.

The Brains Trust (in alphabetical order).

Ian Dand, Gregor Macfarlane, Martin Renilson and Giles Thomas;
For their time, humour, patience and for showing me the wood and the trees.
Without any one of them this work would have been impossible

“The teacher is the needle, and the student but the thread”. Miyamoto Mushashi

The AMC / UTAS

For awarding a scholarship to the author to study part time, afterhours, in two different continents, with four supervisors, communicate by Skype and email, in a highly non-linear fashion.

Also to the many AMC staff who directly and indirectly assisted the author in this project, (model makers, experimenters, administrators, caterer’s et. al.)

Liz Robbins

For putting up with the “other woman” for so long.
I promise no more research (this year).

Garfield Barwick

“Laboro per fortitudo ut perficio”

Capt. James Cook FRS RN

To my hero James Cook for his inspiring words;

“Do just once what others say you can’t do, and you will never pay attention to their limitations again.”

VIII
# Table of Contents

Frontispiece ...................................................................................................................... I  

Declarations ....................................................................................................................... II  
Statement of Originality .................................................................................................. II  
Authority of Access ........................................................................................................ II  
Copyright ......................................................................................................................... II  

Statement of Contribution ........................................................................................... III  

Abstract ........................................................................................................................... VI  

Acknowledgements ......................................................................................................... VIII  

Table of Contents .......................................................................................................... IX  

List of Figures ................................................................................................................. XII  

List of Tables ................................................................................................................ XV  

Nomenclature ................................................................................................................ XVI  

Abbreviations ................................................................................................................ XVIII  

1. General Introduction ............................................................................................... 1  
   Background ................................................................................................................... 1  
   Statement of the Problem .......................................................................................... 10  
   Approach .................................................................................................................... 10  
   Thesis Structure ........................................................................................................ 11  

2. Experimentation ...................................................................................................... 14  
   Physical Testing ......................................................................................................... 14  
   Data Processing ........................................................................................................ 21  

3. Wash Decay ............................................................................................................. 25  
   Paper Abstract .......................................................................................................... 26  
   Introduction ............................................................................................................... 26  
   Physical Tests .......................................................................................................... 26  
   Wash Measurement ................................................................................................. 27  
   Results ....................................................................................................................... 29  
   Concluding Remarks ............................................................................................... 33  

IX
# 4. Divergent Wave Angle

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Abstract</td>
<td>34</td>
</tr>
<tr>
<td>Introduction</td>
<td>35</td>
</tr>
<tr>
<td>Physical Tests</td>
<td>35</td>
</tr>
<tr>
<td>Wash Measurement</td>
<td>35</td>
</tr>
<tr>
<td>Wave Theory</td>
<td>35</td>
</tr>
<tr>
<td>Results</td>
<td>38</td>
</tr>
<tr>
<td>Future Work</td>
<td>41</td>
</tr>
<tr>
<td>Concluding Remarks</td>
<td>42</td>
</tr>
</tbody>
</table>

# 5. Wash Unsteadiness

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Abstract</td>
<td>43</td>
</tr>
<tr>
<td>Introduction</td>
<td>44</td>
</tr>
<tr>
<td>Physical Tests</td>
<td>44</td>
</tr>
<tr>
<td>Experimental Results</td>
<td>51</td>
</tr>
<tr>
<td>Discussion</td>
<td>64</td>
</tr>
<tr>
<td>Concluding Remarks</td>
<td>67</td>
</tr>
</tbody>
</table>

# 6. Wavelet Analysis Characterisation

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Abstract</td>
<td>68</td>
</tr>
<tr>
<td>Problem Introduction</td>
<td>69</td>
</tr>
<tr>
<td>Signal Analysis</td>
<td>69</td>
</tr>
<tr>
<td>Physical Tests</td>
<td>73</td>
</tr>
<tr>
<td>Results</td>
<td>74</td>
</tr>
<tr>
<td>Discussion</td>
<td>90</td>
</tr>
<tr>
<td>Concluding Remarks</td>
<td>91</td>
</tr>
<tr>
<td>Further Work</td>
<td>91</td>
</tr>
</tbody>
</table>

# 7. When Is Water Shallow?

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paper Abstract</td>
<td>92</td>
</tr>
<tr>
<td>Introduction</td>
<td>93</td>
</tr>
<tr>
<td>Shallow Water Vessel Performance</td>
<td>96</td>
</tr>
<tr>
<td>Wash Performance</td>
<td>101</td>
</tr>
<tr>
<td>Wash Characterisations</td>
<td>102</td>
</tr>
<tr>
<td>Characterisation Summaries</td>
<td>107</td>
</tr>
<tr>
<td>Concluding Comments</td>
<td>110</td>
</tr>
</tbody>
</table>

# 8. Summary, Conclusions and Further Work

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>111</td>
</tr>
</tbody>
</table>
1. **General Introduction**

The thesis title; “Shallow Water Catamaran Wash – Simple Characterisations for a Complex Phenomenon” reflects the two main tenets of this work.

Firstly, it presents a detailed investigation into catamaran generated wash in shallow water, specifically within the trans-critical zone. Critical conditions arise when a vessel moves at the so-called “critical speed” in a given depth of water. As is demonstrated in the thesis, vessels travelling within this zone can give rise to unique phenomena within its wash, which can cause significant safety-related issues for other waterway users and to the environment.

Secondly, the goal of this study was to provide practical and easily-understood wash characterisations which can be easily applied by naval architects, operators and regulators to real-world scenarios. As a result, the research was not simply an academic exercise, but a route toward a practical method for assessing vessel wash.

The academic challenge herein lies in distilling simple characterisations for common use from what is a complex and highly non-linear problem.

**Background**

As commercial road and railway traffic increases, with existing capacity unable to keep up with demand, the inevitable outcome is congestion and gridlock on land. To ease this pressure, governments and private industry are increasingly turning to “blue” or “marine highways” to mitigate the problem, [1].

These marine highways are essentially “soft infrastructure”, which are desirable due to their readiness, large carrying capacity and relatively low capital cost [2], (i.e. ships and ports). Many major cities and towns are situated on waterways and have the potential of taking advantage of these marine highways [3].

However, the impacts of increased marine traffic are significant and must be taken into consideration, specifically those of vessel wash. Such wash concerns take the form of environmental degradation and risk to public safety [4, 5, 6, 7, 8, 9, 10].

In essence, the action of shallow water wash on a shoreline or river/canal bank can be erosive, especially if the wash wave breaks. Risk to public safety can also be compromised if
people are caught by these waves either on shore or in a boat. As a result, wash issues can be as important in vessel design as deadweight and speed.

Ideally a way must be determined to mitigate the environmental and public safety concerns of vessel wake, enabling these marine highways to be utilised sustainably. This must be done before potentially serious and long-lasting damage is done to the environment and other users. However, before mitigation strategies can be established, a full and comprehensive understanding of the problem is required.

Vessel-Generated Wave System

All vessels travelling at or near the water’s surface will create waves in the form of a wave pattern. This system is due to the pressure differences created along the vessel as it moves forwards. In simple terms, this creates a ‘wave resistance’ which, in combination with the frictional resistance, makes up the total vessel resistance [11, 12, 13].

In deep water, the wave system comprises divergent and transverse wave systems, (see Figure 1) which travel with the vessel. The transverse system travels at 90° to the vessels sailing line and the divergent system at 19°28’, (i.e. the Kelvin angle). The vessel’s bow and stern each create their own wave systems, with that from the bow dominating [14].

The deep water wave system is regular and predictable, and in turn relatively simple to calculate or measure physically. However, as a vessel enters shallower water, (at constant speed), boundary conditions change and becomes more difficult to calculate [15] or to measure. The wave pattern changes progressively, with the divergent bow wave angle gradually widening to a maximum of 90° and the transverse wave system disappears, [16].

Figure 1 – Kelvin Deep Water Wave Pattern [17]

At the speed of maximum bow wave angle, (also known as the “critical” speed), a unique non-linear hydrodynamic phenomenon occurs, which is the generation of a solitary wave or
1 General Introduction

“soliton” [18]. Solitons are unique in that they have no preceding or following trough, travel at a constant speed (faster than the vessel) and are undiminished in form over time. The restriction of water depth, (a common feature of inland waterways, ports and harbours), may lead to the development of solitons, for vessels travelling at or near critical speed [19].

Furthermore, a wave generated in deep water may propagate into shallow water, where its characteristics can change significantly. These waves have been shown to have catastrophic effects on both life and property [20].

Clearly the shallow water wave system is more complex than the deep water system. Some shallow water waves in the trans-critical regime are non-linear, time-dependant and fundamentally more difficult to measure [21, 22]. If the goal of utilising marine highways sustainably is to be achieved, a clear understanding of shallow water wash is required.

For a vessel travelling within a channel (or canal) the wave pattern is affected due to the proximity of the side boundaries, (in addition to the proximity of the bottom). The pattern is further complicated by boundary reflections. Recognising that vessel shallow water wave patterns in canals are a large stand-alone topic, it has not been covered within this thesis.

Wave Shoaling

As a wave enters shallow water the dynamics of the wave change; wave speed and wave length decrease and, following the law of conservation of energy, the wave height, (i.e. steepness), increases. Within wave mechanics this process is known as “shoaling” [23]. As the shoaling wave progresses towards the shore, the wave reaches a critical height / steepness, becomes unstable, then breaks forwards, (wave breaking). Wave breaking usually occurs when the ratio of wave height to water depth reaches unity.

Of significance is the shape of the breaking wave which is a direct function of the slope of the bottom. Horikawa [24], described three wave shapes / types: spilling breakers (low bottom slope), plunging breakers (moderate slope), and surging breakers (high slope). Figure 2, (Horikawa), illustrates this.
Accordingly the combination of beach slope and oncoming wash characteristics has a direct impact on shoregoers and the environment.

**Environmental Impact**

As mentioned above, marine traffic in coastal or inland waterways has two immediate effects on the environment: bank erosion and fine sediment re-suspension. This environmental impact has led to new “green” criteria being placed on vessel design and operation [25, 26]. Environmental impact is no longer seen as of secondary importance, but in some cases has become the prime concern and a contractual requirement [27, 28].

Wash related environmental degradation has been reported in many countries, particularly those with high speed marine traffic [9]. Many older traditional waterways, (e.g. Thames, Danube, Elbe, Rhine and Volga), have already been significantly affected, with their current environment being changed significantly from the original and now are mostly armoured [29].

The environmental impact of wash manifests itself primarily as the erosion of shoreline or river banks, and in turn causes the degradation of ecosystems. Vessels introduce “foreign”, man-made waves into the environment which has previously been only exposed to naturally occurring wind waves and tide. Vessel waves often have a height, period and energy profile vastly different to the naturally occurring wind waves. The potential exists that, under the stress of these new waves, significant changes will occur to the environment, which may be irreversible [30].

A hidden effect of wash is the re-suspension of fine sediments. Such re-suspension is significant in that it makes breathing increasingly difficult for fish and insect larvae alike, whose habitat is normally fine sediment free. Additionally the increased turbidity reduces the light availability essential to the growth of plankton and plant species [31].
Many areas which receive significant wave activity experience little or no negative effects due to the pre-existing natural wave action, typically from ocean swell and tides. It is the relative sensitivity of a shoreline or bank to erosion that is important to determining wash impact. An ecosystem’s vulnerability to degradation is a function of wash characteristics, topography, sediment type, sediment size and tide [32, 33]. The degradation typically consists of progressive habitat destruction via shore and bed erosion. Marine life will be directly affected by damage or loss of marine ecosystems.

In addition to the environmental degradation of sensitive waterways, there may also be damage to the “man-made” environment (e.g. wharfs, jetties, docks and breakwaters) [34, 35, 36]. While such infrastructure damage is reversible (i.e. repairable), there is a clear and, possibly considerable, associated economic cost. There are also the legal implications of who is to blame and who must be recompensed.

The Oxford dictionary defines pollution as; “the introduction into the environment of a substance which has harmful or poisonous effects”. As no identifiable benefit arises from wash, it can be identified as a form of vessel generated pollution.

Wash pollution can be considered in the same vein as noise pollution, with both typically being cyclical transient wave forms. Its effects are very much dependant on amplitude, frequency, (i.e. energy) and exposure time [37]. While there is common understanding and awareness of noise pollution, (and its effects), there is little for vessel wash. This may be because its impact is often only indirectly felt by humans.

The United Nations Convention on Law of the Sea [38], (UNCLOS), is a comprehensive document stating the rights and responsibilities of nations with respect to the world’s oceans. It covers navigational rights, economic rights, scientific exploration, piracy and importantly pollution of the seas. UNCLOS Part XII - Protection and Preservation of the Marine Environment, specifically deals with vessel borne pollution and the environment. Some key articles are quoted below.

Article 192: “States have an obligation to protect and preserve the marine environment.”

Article 194: “States shall take...all measures consistent with this Convention that are necessary to prevent, reduce and control pollution of the marine environment from any source...” “The measures taken in accordance with this Part shall include those necessary to protect and preserve rare or fragile ecosystems as well as the habitat of depleted, threatened or endangered species and other forms of marine life.” Article 211 encourages member states to pass laws and regulations governing pollution from not only their own ships, but also foreign flagged ships entering their waterways.
While vessel wash is not specifically mentioned within UNCLOS, the intent of the document is clear with respect to pollution and hence vessel-generated waves by inference.

**Vessel Type**

It is not simply the actual and potential increase in vessel traffic which has brought wash concerns to the fore, but also the size and speed of the vessels themselves. Vessel attributes, (i.e. hull configuration, length, displacement, configuration and speed), have a significant effect on the type of wash generated.

The combination of high-speed, lightweight diesel engines and high strength-to-weight materials (i.e. aluminium and composites), has resulted in recent years in a significant increase in vessel speed and deadweight. Such vessels, commonly referred to as high speed craft (HSC), are predominantly catamarans [39], but are also planning monohulls. The advent of HSC’s are a clear improvement on an operational and economic basis, however they can be severely detrimental to the environment.

With respect to hull configuration, it should be noted that non-planning monohulls generally operate over a large displacement (deadweight) range and low length-beam ratios, while planning monohulls and catamarans operate within a narrow displacement range and have larger length-beam ratios (slender). It follows that their wash in shallow water are significantly different [40, 41].

As a vessel approaches the “critical speed”, (i.e. in depth restricted water), its resistance increases significantly [42]. However HSC, (due to their high power-to-weight ratios), can sustain the critical speed for longer, which in turn encourages soliton development. So a technological barrier to craft operating at near-critical speeds has been removed.

The sizes of merchant ships and conventional cargo vessels have also increased due to advances in shipbuilding technology. While ship dimensions have increased, the dimensions many of the waterways they use have not. In turn the average depth/draught ratio, (or, alternatively, underkeel clearance), has decreased significantly. This is not a problem with respect to the shallow water waves produced by these vessels, due to their relatively low speeds, but is significant with respect to waterway bed erosion due to the scouring effect induced by accelerated flow around the vessel [43].

Furthermore, those vessels with large bluff bows and large hull volumes have a significant swell-up at the bow, followed by a significant trough along most of the hull. The trough of this so-called Bernoulli (or “local”) wave is commonly referred to as ‘drawdown’. The
Bernoulli wave is essentially the dynamic displacement of water due to the vessels forwards motion [17].

The author’s previous post graduate work concentrated on the wash of catamaran HSC in deep water, [44]. This vessel type was specifically selected due to its aforementioned potential for wash generation and its being the predominant form in the high-speed passenger ferry fleets. Accordingly the focus of this new work carries on into shallow water examining the wash generation from HSC catamarans.

Public Safety Impact

As previously discussed, not only can vessel wash affect the environment, but it can also have an effect on other waterway users, (e.g. commercial and recreational vessels, marine infrastructure, swimmers and shore-goers). It can be a minor irritant affecting the enjoyment of others within the littoral zone; however it can also be a significant public safety hazard.

The effect of wash on a vessel is dependent on the characteristics of the wash itself, (i.e. wave period, frequency, heading and water depth). A free floating vessel’s dominant response to waves is in the motions of pitch, roll and heave, (see Figure 3). Additionally the orientation of the target vessel to oncoming wash has a major impact on outcomes, with beam-on waves inducing roll, and head-on waves inducing pitch [42].

![Figure 3 – Vessel Seakeeping Responses](image)

Each vessel type has its own particular worst case harmonic depending on wave characteristic [17]. Typically broaching and capsizing are more of a problem for smaller vessels, with larger vessels seeing high rolling, potentially leading to unintentional course changes.

In general, commercial vessels are large vessels, crewed by professionals; recreational vessels are smaller vessels, crewed by amateurs. It could be argued that recreational vessels are
afloat for recreational reasons rather than commercial ones, and it can be assumed that they are in a relaxed state and not focussing on the possibility of a severe wash event.

While wash poses a threat to free-floating vessels, the risk to berthed vessels can be higher. Moored vessels are usually asymmetrically restrained by mooring lines along one side. These lines heavily restrict the vessels response to oncoming wash, and can lead to slamming and breakage of the vessel and dock. This can prove especially significant if the vessel is in the process of loading passengers or cargo.

Additionally vessels passing moored vessels can induce significant surge, sway and yaw motions due to the interaction between the two vessels [45]. This interaction can also lead to mooring line failure and vessel damage. The Bernoulli wave dominates in some shallow water scenarios, being the main driver of ship-ship interaction for moored and free floating vessels alike [46].

There has been one recorded death which can be directly attributed to vessel wash; the Purdy Incident [6]. A 10m long monohull fishing vessel, Purdy, was swamped by HSC wash. A crew member was washed overboard and subsequently lost at sea. This was despite the weather being fine and clear, the vessel being headed into the oncoming wash and speed restrictions being placed on the HSC. The official accident report suggested that a soliton may have been to blame, which passed across shoaling water, thereby increasing in size as it approached the Purdy. This incident highlighted the possible extreme danger and unpredictability of wash.

As aforementioned – as deep water waves enter shallow water, they begin to “feel” the bottom, which translates to an increase in height and a corresponding reduction in wave speed [47]. This phenomenon continues until the waves finally break on the shore. Shore-goers regularly experience wind waves (i.e. swell) which exhibit this behaviour, known as “surf”. Vessel-generated waves exhibit the same wave shoaling phenomenon, but unlike swell, wash is often not regular or predictable, (particularly if solitons are involved).

Many recreational shore users are day trippers, who are likely to be unfamiliar with the local sea conditions and littoral environment. Also such day trippers are at the sea side for recreational reasons rather than commercial ones, and, (similarly to recreational vessels), it can be assumed that they are focused on leisure activities rather than vessel wash.

One might assume that a HSC passing in the distance would have little effect on shore-goers, due to the expected dissipation of the vessel’s wash with distance. However if said vessel is operating near the critical area a soliton may be generated, and shed [18]. The effects of such a solitary “rogue wave” will be felt long after the vessel has disappeared from view [48]. Solitons travel large distances without significantly diminishing, and also travel faster than
expected (faster than the vessel which generated them) [19]. In turn this wave may arrive sooner than expected and also with a much greater energy than conventional wash [49].

The combination of solitons, recreational vessels, and recreational shore-goers poses a significant safety risk, especially for popular beaches near a coastal HSC route.

Mitigation
The public safety risk, as well as damage to infrastructure, has led authorities to implement mitigation strategies. These have typically taken the form of route planning and assessment, speed restrictions, access restrictions, and shore warnings [50, 26], (see Photo 1).

These measures may have had the desired mitigating effect, but have not necessarily increased understanding of the phenomena. For instance blanket speed restrictions, (i.e. 5 knot go-slow areas), are not an accurate reflection of current understanding of shallow water wake. For example some vessels, particularly planning craft, can create larger waves when operating at slower speeds.

Metrics / Characterisations
There are multiple measures currently utilised to assess vessel wash, (i.e. wave height, wave period, decay, direction, energy, energy flux, propagation, wave shape etc.), [25, 27, 28, 47, 51, 52, 53, 54, 55]. Furthermore there is lack of accord of which measure to utilise, (and even disagreement of the method used to measure individual metrics).

Part of the lack of understanding of shallow water wash may be the aforementioned measures and methods are not suitable when applied to shallow water.

Each measure alone does not provide significant or comprehensive understanding of the wash, (as one measure should). What is required is a measure from which the potentially damaging aspects of the wave can be clearly identified. The goal will be to create a wash measure, which will be a robust and accurate indicator for comparison between vessels and their wash.

Summary
It has been established that; vessel wash is a significant risk to public safety and to the environment; vessel wash should be re-considered as a form of pollution; shallow water wash needs closer examination due to its increased risk; further work, (research), is required to clarify this complex phenomenon; simple characterisations should be developed to encapsulate this understanding.
Statement of the Problem

As prefaced within this thesis' title and the above introduction, the problem can be separated into two parts, being also the global project goals;

(a) To define clearly the features of “Shallow Water Catamaran Wash” and;

(b) To describe and condense these “Complex Phenomena” into “Simple Characterisations”.

Approach

The following methodology has been utilised to solve the project goals.

Literature search

A literature search was completed by studying academic publications, subject textbooks, bibliographies, key texts, reference lists, library databases and the internet. From this the following was determined;

Theoretical work on deep water wave generation and propagation is well-established and understood. The majority of existing works only provide a very general and limited understanding of the problem. The major wash studies; Ship Wash Impact Management (SWIM 2000), and Safe Passage and Navigation (SPAN 1999) highlight the risk (outcomes) of vessel wash on other vessels, marine structures, beach-goers and the environment, but fall short in their capacity to characterise vessel wash fully.

Post literature search the following key questions remained; “How does deep water wash differ from shallow water?”, “Can the same deep water wash characterisations be used for shallow water wash?” and “When does is shallow water occur?” As expected a literature search alone was not sufficient to answer the project questions and experimentation was required.

The literature review has been presented in a broad sense within this chapter and more topically within each of the five papers. It should be noted that the references are presented sequentially within this thesis.

Experimentation

The experimentation methodologies available were based on physical and numerical testing.

Physical test options comprised model and full scale testing. Model scale testing was selected as the only a viable physical test method, with full scale tests being prohibitively expensive.
and time consuming. Model scale testing features a controlled environment providing consistent and reproducible results. The wash generated by the vessel at each condition could be accurately measured, recorded, and analysed. Model testing is not as onerous as full scale testing, nonetheless involves significant resources, (i.e. a wide model test basin, measurement equipment, model makers).

Numerical test options comprised of classical numerical methods (simple approximating integrals) or modern computational fluid dynamic (CFD) codes. Both methods are desk based, utilising commercially available computer programs, and provide a quicker and more cost-effective method than physical test methods.

CFD was selected as it is a more comprehensive calculation method compared to the classical methods. From the author’s previous M.Phil work, utilising Shipflow [56], it was understood that CFD can successfully calculate deep water wash. However it is unable to predict absolute values, but can give trends, which are good indicators of a vessel’s wash characteristics.

A combination of CFD testing, supplemented with physical model scale testing, was proposed. This was believed to be the best blend of experimental methods, providing a large volume of high quality (repeatable) results, which had been validated (albeit in deep water) in the test tank.

Preliminary CFD runs were encouraging. However, as tests moved into increasingly shallow water, the program was unable to converge to a solution which provided a sensible result. Guidance from the program’s developer (Flowtech) indicated that, (at that time), the code was unable to take into account the highly non-linear fluid dynamics in such water depths, and so may not be suitable for such research.

It was therefore decided that shallow water CFD experimentation was unreliable and it was discontinued in favour of the more time-consuming, but reliable, scale model testing.

**Thesis Structure**

Due to the part-time nature of the study programme a “thesis by publication” strategy and format was adopted. This format comprises academic papers forming discreet thesis chapters, with framing text introducing the research (below) and linking each of the chapters and papers, into a unified body of original work. As for a conventional format, the outcome is a sustained and cohesive theme which is maintained throughout the document.

The place of publication is important in establishing the academic value of the published papers. The journal selected was the Royal Institution of Naval Architects International

As prefaced, the global project goals are; (a) to define clearly the features of shallow water wash and; (b) to describe these features by simple characterisations. The outcomes of each of the papers meet either one, or both, of these project goals.

**Paper 1 - Wash Decay (Chapter 3)**
The goal of Paper one was to define the wave decay coefficient (n) for shallow water. It was believed that (for a given speed) this coefficient varied significantly with change in water depth. Therefore it could be utilised as wash characterisation. Additionally the effect of displacement and water depth were investigated.

An unexpected outcome from this paper was that the use of the wave of maximum amplitude (\(H_w\)), was shown to be unsuitable for shallow water. In its place, the leading wave height (\(H_{lead}\)) proves to be a more suitable parameter.

Following on from observations made during the physical test programme, it was believed that the leading wave angle could be utilised to characterise vessel wash in shallow water.

**Paper 2 - Divergent Wave Angle (Chapter 4)**
The goal of Paper two was to define the leading wave angle in shallow water. As for the decay coefficient, it was believed that (for a given speed) the leading wave angle varied significantly with change in water depth. In turn it could be utilised as a wash characteristic. Additionally the effect of hull form and h/L ratio on the leading wave angle was investigated.

An unexpected outcome from this paper was that wave growth was recognised as a possible feature of shallow water wash, especially around the critical number.

**Paper 3 - Wash Unsteadiness (Chapter 5)**
The existence of wave growth raised concerns with regard to confidence associated with the accuracy of existing results. The goal of paper three was to investigate and define wave growth in shallow water. It was believed that unsteadiness was a feature on the “near-critical” shallow water operational zone, and (once defined) could be utilised as a wash characteristic.

An unexpected outcome from this paper was that, in addition to wave growth, the occurrence of solitons can be further utilised to characterise shallow water wash.
Paper 4 - Wavelet Analysis Characterisation (Chapter 6)
It was recognised that the shallow water wash characterisations presented in papers one, two and three, (in addition to the existing characterisations), were simplistic and not fully descriptive of this dynamic region. From a review of each characterisation it was realised that the factor common to each was the longitudinal wave cut. It was further realised that a longitudinal wave cut can be considered as a transient signal and in turn it may be open to signal analysis techniques. Specifically it was believed that wavelet analysis would provide a more holistic characterisation of shallow water wash.

The wavelet analysis method is not in keeping with “simple characterisation” ethos; however the resultant 2D and 3D graphical plots are simple, yet powerful, characterisations. This technique is a “new and novel” approach to an existing problem and also meets both the project goals.

Paper 5 - When Is Water Shallow? (Chapter 7)
Papers one, two, three, and four define key features of shallow water wash. To accentuate the relevance of these characterisations it was necessary to place them within the context of established vessel performance-related characterisations. Paper five can be described as a generalised summary paper of shallow water vessel performance characterisations.
2. Experimentation

Physical Testing

As stated in Chapter 1, a physical scale model test programme was adopted in order to meet the global goals. This programme consisted of a model test facility, scale models and data acquisition equipment. It should be noted that for the avoidance of repetition within each of the chapters (papers), the physical testing description has been removed and referenced back to this chapter. Additionally, for the sake of brevity, specific run data and photographs have been placed in the appendix.

Model Test Facility

The facility utilised for the physical model testing was the Australian Maritime College's (AMC) Model Test Basin (MTB). It is 35 m long, 12 m wide and has a water depth range of zero to 1.0 m. It is fitted with an electric winch system capable of towing light models (<40 kg) up to a maximum speed of approximately 3.75 m/s and a multi-segment wave maker.

The bottom of the basin is flat providing the ability to conduct experiments in very shallow water depths. The combination of large width (compared to a towing tank), solid flat bottom, variable water depth, model towing winch and wave generation capability makes this facility very versatile. Wave damping materials were added along the tank walls to minimise wave reflections.
Instrumentation

The two key instrumentation / systems utilised were the main towing winch system and the wave probe system. The speed of the model was determined using an encoder connected to the electrically driven winch drum. This system was regularly calibrated by a time on known distance method.

The wave probes utilised were 300mm long capacitance type hard-wired to a power supply and signal conditioner supplied by Hydraulics Research Wallingford (HRW). Each probe’s signal was amplified to obtain an optimum signal resolution for the expected wave height, which varied significantly with speed. At the start of each test condition, (i.e. water depth, model or displacement change), and at the beginning of each day, each of the probes was calibrated.

Test Procedure and Data Acquisition

The following procedure was the same for all water depths tested. Firstly the initial instrumentation readings were recorded and subsequently checked between runs. The carriage speed was set and the run began. The carriage accelerated and data acquisition began at a (constant) set point. The equipment recorded for approximately 35 seconds, at a sample rate of 200 Hz. After the model had passed a set point it was decelerated to a stop. The tank was allowed to settle to a predetermined level, and the entire process repeated.

Test Programmes

A total of three test programmes were conducted at the AMC MTB; Programme 1 in March 2006; Programme 2 in January 2007; and Programme 3 in February 2009. The focus of each programme reflected each published paper’s goal. A summary of the test programmes is shown in Table 1 with the full programmes detailed in the Appendix.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Programme 1</td>
</tr>
<tr>
<td>V (m/s)</td>
<td>0.68 - 3.60</td>
</tr>
<tr>
<td>Frh</td>
<td>0.50 - 2.10</td>
</tr>
<tr>
<td>FrL</td>
<td>0.14 - 0.62</td>
</tr>
<tr>
<td>h (m)</td>
<td>0.19 - 0.38</td>
</tr>
<tr>
<td>h/L</td>
<td>0.075 - 0.150</td>
</tr>
<tr>
<td>Hull</td>
<td>EH &amp; NPL+</td>
</tr>
</tbody>
</table>

Table 1 – AMC MTB Physical Test Programmes
Probe Arrangement
An array of wave probes was utilised to ensure detailed and accurate measurement of the generated wave patterns. The probes were mounted either on an aluminium beam protruding from the tank side, or, free standing within the tank. The number and arrangement of the probes was dependant on the test programme. The main longitudinal probe array was offset a nominal 2L from the sailing line, which was in keeping with previous deep water MTB tests. The probe spacing was set at whole multiples of the model length. For each run the model was started at the same position, with the model acceleration rate varied depending on the desired final speed.

For Programme 1 an array of eight probes was arranged to record the wave pattern. Five probes were attached to a transverse mounting arm and three were free standing in the tank, see Figure 4. The probes were offset at one metre intervals from the sailing line, equivalent to 0.4L. The main transverse array measured wave height, decay (as a function of transverse distance from the sailing line), and leading wave angle, (probes 1, 2, 3, 4 and 5). The longitudinal array measured wave "growth", (probes 6, 16, and 17). Probe 8 was used to check for blockage and any asymmetrical effects, (which were later shown to be negligible).

For Programme 2, so as to more accurately capture the wave pattern, the number of probes on the transverse arm was doubled (one every 0.5m). As for Programme 1, the main transverse array measured wave height and leading wave angle, (probes 1, 2, 3, 4, 5, 6, 7, 8, and 9), and the longitudinal array measured wave "growth, (probes 10, 9, and 11). Following inconclusive results, the probe utilised to check for blockage and any asymmetrical effects was removed.

For Programme 3, the number of probes was increase even further, primarily to capture longitudinal wave growth. Additional transverse arms were added, and the longitudinal array was extended. In total some seventeen probes were utilised, see Figure 6. The main transverse array measured wave height, decay (as a function of transverse distance from the sailing line), and leading wave angle, (probes 1, 2, 3, 4, 5, 6, 7, 8 and 9). The secondary transverse arrays measured leading wave angle only, (probes 12 + 13 and probes 10 + 11). The longitudinal array measured wave "growth", (probes 15, 16, 12, 17, 9, 18 and 9).

As the number of probes increased, the time taken to conduct each calibration became a significant factor due to the number of probes and water depths changes.
2 Experimentation

Figure 4 – Programme 1 MTB Probe Arrangement

Figure 5 – Programme 2 MTB Probe Arrangement
Figure 6 – Programme 3 MTB Probe Arrangement

Photo 3 – Programme 2 Probe Array
2 Experimentation

Hull Forms
Two hull forms were tested, one a low wash type, the other a conventional fuller form. The hull parameters for both models can be seen in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>EH</th>
<th>NPL+</th>
</tr>
</thead>
<tbody>
<tr>
<td>L (m)</td>
<td>2.50</td>
<td>2.50</td>
</tr>
<tr>
<td>( \Delta ) (kg)</td>
<td>23.16</td>
<td>51.28</td>
</tr>
<tr>
<td>( \frac{L}{\sqrt{3}} )</td>
<td>11.14</td>
<td>8.48</td>
</tr>
<tr>
<td>L/B</td>
<td>19.86</td>
<td>11.00</td>
</tr>
<tr>
<td>B/T</td>
<td>1.76</td>
<td>2.00</td>
</tr>
<tr>
<td>( C_p )</td>
<td>0.625</td>
<td>0.693</td>
</tr>
<tr>
<td>S/L</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Trim (static)</td>
<td>0°</td>
<td>0°</td>
</tr>
</tbody>
</table>

Table 2 – Test Hull Form Parameters

NPL + Hull Form
The National Physical Laboratory (NPL) series is based on a round bilge hull form designed for high speed, \( (Fr_L 0.3 – 1.2) \). The series was developed by Bailey [57] of the NPL (Ship Division) as a preliminary resistance and propulsion design tool for work boats, fast patrol craft and small naval ships.

Molland and Lee [58] utilised the NPL series in an investigation into the effect of prismatic coefficient on catamaran and monohull resistance. The hulls had the same \( C_p, C_b \) and LCB as the parent hull form NPL 100A, but with differing L/B and B/T ratios, which were more typical of modern catamaran hull forms. The hull form is referenced here as the NPL+.
EH Hull Form

The EH or Energy Hysteresis hull form tested is the parent hull of a series previously tested at the AMC, [59]. This series was developed to have low wash characteristics, be commercially viable, and 'scalable' within a systematic series.
2 Experimentation

Data Processing

As described, the wave data was derived from longitudinal cuts taken though the free surface at given offsets from the sailing line the wave cut data simply being a record of free surface elevation (H) against time (t). For each run there are multiple longitudinal cuts, each for a given transverse offset from the vessel sailing line.

The data post processing comprised of three stages. Firstly the raw data analysed with a high pass filter to remove any background signal noise. The second stage is to check plot each run to ensure the data record is complete. The third stage (for decay analysis) the text data file is imported into spread sheets for wave angle / decay / growth analysis, (Figure 9, Figure 10 and Figure 11). The third stage (for spectral analysis) the text data file is imported in into Matlab for wavelet analysis.

Figure 8 - EH Body Plan
### Zone 1

<table>
<thead>
<tr>
<th>Maximum occurs at</th>
<th>$H_W$ (mm)</th>
<th>Period (s)</th>
<th>From</th>
<th>To</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.41 (mm)</td>
<td>11.25</td>
<td>6.28</td>
<td>10.80</td>
<td>10.70</td>
</tr>
<tr>
<td>-3.85 (mm)</td>
<td>11.53</td>
<td>0.56</td>
<td>11.70</td>
<td>11.70</td>
</tr>
</tbody>
</table>

### Zone 2

<table>
<thead>
<tr>
<th>Maximum occurs at</th>
<th>$H_W$ (mm)</th>
<th>Period (s)</th>
<th>From</th>
<th>To</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.62 (mm)</td>
<td>13.38</td>
<td>12.54</td>
<td>12.60</td>
<td>12.60</td>
</tr>
<tr>
<td>-6.81 (mm)</td>
<td>13.10</td>
<td>-0.56</td>
<td>14.00</td>
<td>14.00</td>
</tr>
</tbody>
</table>

### Zone 3

<table>
<thead>
<tr>
<th>Maximum occurs at</th>
<th>$H_W$ (mm)</th>
<th>Period (s)</th>
<th>From</th>
<th>To</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.53 (mm)</td>
<td>15.76</td>
<td>13.38</td>
<td>15.40</td>
<td>15.40</td>
</tr>
<tr>
<td>-5.85 (mm)</td>
<td>16.02</td>
<td>0.52</td>
<td>19.80</td>
<td>19.80</td>
</tr>
</tbody>
</table>

---

Figure 9 – Wave Analysis Spreadsheet -Wave Cut Page

22
### Figure 10 – Wave Analysis Spreadsheet - Wave Angle Page

#### Table 1: Wave Angle Analysis

<table>
<thead>
<tr>
<th>Speed (m/s)</th>
<th>Zone 1</th>
<th>Zone 2</th>
<th>Zone 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.238</td>
<td>Wave Angle</td>
<td>20.20°</td>
<td>Wave Angle</td>
</tr>
<tr>
<td>Offset (m)</td>
<td>H&lt;sub&gt;w&lt;/sub&gt;</td>
<td>@ Time (s)</td>
<td>Dist (m)</td>
</tr>
</tbody>
</table>

#### Wave Angles

- **Zone 1**: Wave angle varies from 20.20° to 19.03° with an average of 19.75°.
- **Zone 2**: Wave angle varies from 19.03° to 18.97° with an average of 19.00°.
- **Zone 3**: Wave angle varies from 18.97° to 19.00° with an average of 19.00°.

The wave angles show a consistent decrease in angle from Zone 1 to Zone 3, indicating a potential change in wave direction or wave type as the distance increases from the starting point.
## Experimentation

![Graph showing Leading Wave Decay](image)

**Figure 11 – Wave Analysis Spreadsheet -Wave Decay Page**
3. Wash Decay

This research was originally published as;

“The Decay of Catamaran Wave Wake in Shallow Water”

Alex Robbins¹, Giles Thomas¹, Gregor Macfarlane¹, Martin Renilson¹, Ian Dand²

1, Australian Maritime College, Tasmania, Australia.
2 BMT SeaTech Ltd, Southampton, UK.


For the avoidance of repetition the original paper has been modified for this thesis.
Paper Abstract

The work presented characterises the effect of water depth on catamaran wash, specifically wave height, and the rate of decay coefficient. From model test results a simple and novel method for the prediction of catamaran wash, across the trans-critical Froude depth range is presented. The experiments showed that the decay coefficient of the vessel-generated waves varies with Froude depth number.

Introduction

Traditionally vessel wash has been of secondary concern to other design parameters such as speed, fuel consumption, and deadweight. However, as waterways previously only open to low volumes of traffic are being utilised by larger numbers of vessels, the environmental impact of vessel wash is becoming increasingly important. As a consequence, new criteria are being placed on these vessels, such as meeting specific wash height requirements.

Two hull forms were chosen to investigate the effect of water depth on vessel wash. The first was typical of a low wash high speed catamaran, and the second was representative of a fuller, more traditional, higher displacement catamaran. These hull forms were tested in three water depths and at a number of speeds, covering the trans-critical and super-critical Froude depth number range.

The purpose of the tests was to determine a means whereby the wave height of a fast catamaran can be estimated easily from basic hull parameters. Such a method would enable a designer to determine whether environmental goals for wave height some distance from the sailing line could be met. An important part of this quest is to determine how the wave system decays with distance from the sailing line.

The data from the model tests were analysed using a decay type method, [53]. It is known that the wash from a vessel in deep water, along the cusp line, decays at a rate approximately equal to the inverse cube root of distance from the sailing line. It is also known that the decay rate can vary greatly from this approximation in shallow water, [60]. In addition, vessel wave heights in shallow water have been shown to be functions of Froude depth number and h/L ratio, [61].

Physical Tests

Two hull forms were tested, one a low wash type, the other a conventional fuller form. They tests were carried out in shallow water at the AMC MTB. The model test basin, the test programme, measurement and data acquisition, and hull forms utilised have been fully described in Chapter 2, and for the sake of brevity will not be repeated here.
3 Wash Decay

Wash Measurement
As the issue of wash has become increasingly important, so too has the need to have a
method by which to assess it. Currently, there is no accepted standard for wash assessment.
But there are criteria currently utilised in commercial tenders to measure a vessel’s wash
characteristics.

The suitability of a proposed wash measure was described by Macfarlane and Renilson [52].
A suitable measure should be:

i. independent of location;
ii. independent of sample size;
iii. easy to understand;
iv. easy to measure; and
v. able to rank a vessel’s wake performance.

The measurement method utilised in this work follows these guidelines.

Wave Decay Function
It is known from Havelock [62] that in deep water wave amplitude on the cusp line of the
divergent wave system decays at a rate approximately equal to the inverse cube root of the
distance from sailing line, equation (1). Renilson and Lenz [53] took advantage of this to
develop a method of assessing vessel wash which is independent of lateral distance. They
observed that a vessel’s maximum wave height (H_w), can be calculated for a given offset from
the sailing line (y), once gamma (γ), and the decay coefficient (n) in Equation 1 is known.

\[ H_w = \gamma \cdot y^n \]  

Equation 1

From this it is clear that γ is a good measure of the size of the vessel generated waves in deep
water, which is independent of lateral distance from the vessel centreline.

To obtain γ for a vessel at a particular speed, a series of longitudinal wave cuts must be taken
at various offsets from the vessel. The cuts are analysed for maximum wave elevation, and
these maxima are plotted as a function of y. From this plot a curve of best fit is obtained, and
γ determined.

Figure 12 is a plot of deep water data fitted with a decay curve utilising a deep water decay
coefficient, n = -1/3. It is known that in shallow water the decay coefficient, n, is not always
constant, and varies with vessel speed, and water depth, [63].
Figure 12 - Deep Water Bow Wave Decay Curve [44]

Figure 13 is a plot of shallow water data fitted with a deep water decay curve, \((n = -1/3)\), and also a decay curve, which utilises a decay coefficient for best fit. It can be clearly seen that the deep water decay curve has a poor data fit compared to the curve where the decay coefficient is not set to -1/3.
Results

The raw experimental data was analysed using an Excel spread sheet, which was used to calculate the maximum amplitude within each longitudinal wave cut. The maximum wave amplitude, \( (H_w) \), was the highest consecutive peak and trough in a longitudinal wave cut.

Probe 1 was considered to be influenced by ‘local wash effects’, as described by Macfarlane and Renilson [64], and was not utilised. The four remaining probes, 2, 3, 4 and 5 were utilised in solving for the best fit for \( \gamma \) and the decay coefficient, n, as described in equation (1).

For both hulls and at each \( h/L \), three repeat runs were made, at \( Fr_h = 0.7, 1.0, \) and 1.3. Accordingly the experimental range banding is plotted on Figure 13; this data range was relatively constant for all repeat runs.

Analyses of the wave cuts show significant superposition of competing wave systems. From the typical plots, Figure 14, it can be seen that the position of the maximum wave height changes unexpectedly across the probes. Probes 3 and 5 are clearly affected by an unfavourable superposition of waves, and provide an erroneous result.

![Figure 14 - Maximum Wave Height (Hw) - NPL Model - h/L = 0.150](image)

\[
H_w(bow) = 28.5 \text{ mm} \\
H_w = 45.2 \text{ mm}
\]

\[
H_w(bow) = 27.9 \text{ mm} \\
H_w = 27.9 \text{ mm}
\]

\[
H_w(bow) = 25.6 \text{ mm} \\
H_w = 36.1 \text{ mm}
\]
3 Wash Decay

Whereas the bow wave is clearly identifiable, across all probes, the maximum wave height is not. A plot of the resultant wave angles, Figure 15, further illustrates the problem. This effect was sporadic across the trans-critical depth Froude number range and probe location.

The use of the maximum wave height as the primary measure across the trans-critical range was not possible. Instead the bow wave has been utilised since it is easily identifiable across the trans-critical range, and provides similar decay characteristics.

![Figure 15 - Wave Angle (NPL+ Model - h/L = 0.150 @ Frh = 1.4)](image)

Wave Amplitude

The results from the model tests are plotted in Figure 16 for a single lateral distance corresponding to 2L. Two primary groupings are clear within this plot, being those of the NPL+ model and the EH series model. The heavier (NPL+) model has the greater wave height across the Frh range than the lighter (EH) model. Work done on fast catamarans by Dand et al. [22] found a similar trend.

Within each of the primary groupings, a sub grouping exists of varying h/L values. The lowest h/L (shallowest water), giving the highest $H_w$ values. This is in keeping with other works on catamarans in shallow water; [22], [65], [66], [67].

Gamma ($\gamma$)

The wave amplitude results were analysed to determine gamma. Figure 17 shows a plot of $\gamma$ determined from utilising a variable n. As expected these plots show similar displacement

30
and water depth groupings as discussed above in the Wave Amplitude results. \( \gamma \) reaches a peak value just after the critical \( Fr_h \) number for all cases.

**Variable Decay Coefficient**

Figure 18 is a plot of the variable \( n \) as a function of Froude depth number. The results for both hull forms at each displacement, and water depth tested are shown. A dashed horizontal line representing the fixed deep water coefficient, \( (n=-1/3) \) is shown. Also a fitted polynomial trend line is shown.

The value of the decay coefficient, \( n \), differs from the deep water value of \(-1/3\), and is a function of the Froude depth number for each case. Across the trans-critical and onto the super critical \( Fr_h \) range, the decay coefficient, \( n \), varies with \( Fr_h \), and is not a single fixed value. Also \( n \) appears to be independent of hull form and displacement.

Doctors and Day [60], in a numerical investigation into high speed vessel waves, predicted a similar decay coefficient curve. It was suggested by them that the decay coefficient, across the trans-critical range, could vary from -1.06 to -0.2. From the trend line shown in Figure 18, the decay coefficient has been shown to vary from -1.0 to -0.2.
3 Wash Decay

Figure 17 - $\gamma$ as a Function of Froude Depth Number

Figure 18 - Decay Coefficient for n as a Function of Froude Depth Number
Concluding Remarks

Two hull forms were tested in three water depths and at multiple speeds, covering the trans-critical and super-critical Froude depth number range to investigate the effect of water depth on vessel wash. The data sets from the model tests were analysed using a decay type method, resulting in a simple and novel method for the prediction of catamaran wash, across the trans-critical Froude depth range.

For the hull forms and conditions tested it can be concluded that $\gamma$ is a function of hull form and displacement. In addition the decay coefficient, $n$, appears to be independent of hull form and displacement and is also a function of Froude depth number and for across the trans-critical range may vary between -1.0 to -0.2.

A further review of these results found that some data may have experienced “unsteadiness”. The term “unsteadiness” relates to the variation of a vessel’s (measured) wave pattern over time. This warranted closer investigation and culminated in the work presented in Chapter 5.
4. **Divergent Wave Angle**

This research was originally published as;

“Vessel Trans-Critical Wave Wake, Divergent Wave Angle and Decay”

Alex Robbins¹, Giles Thomas¹, Martin Renilson¹, Gregor Macfarlane¹, Ian Dand²

¹ Australian Maritime College, Tasmania, Australia
² BMT SeaTech, Southampton, UK


For the avoidance of repetition the original paper has been modified for this thesis.
Paper Abstract

Vessel wash in the "deep water" condition, or the sub-critical depth Froude number range, is well understood, trans-critical wash less so, despite vessels frequently operating within this region. The significant effect of water depth on catamaran wash and the fundamental changes in the wave pattern are presented. Specific attention is paid to wave height and to vessel leading wave angle.

The experiments showed that the vessel wave patterns vary with depth Froude number, closely match the theoretical predictions, and that the leading wave angle closely corresponds to numerical predictions. However the peak value is found to occur closer to $F_{nh} = 0.9$ than 1.0.

Introduction

As the effects of vessel wash in deep water become better known it is necessary that the less known effects of shallow water are understood. Accordingly two hull forms were chosen to investigate the effect of water depth on vessel wash. The purpose of the tests was to determine how the wave system, (specifically the bow wave angle), varied across the trans-critical depth Froude range. A secondary goal was to record the effect of hull form and depth – length ratio on bow wave angle.

Physical Tests

Two hull forms were tested, one a low wash type, the other a conventional fuller form. They tests were carried out in shallow water at the AMC MTB. The model test basin, the test programme, measurement and data acquisition, and hull forms utilised have been fully described in Chapter 2, and for the sake of brevity will not be repeated here.

Wash Measurement

The wash measurement method utilised has been fully described in Chapter 3, and for the sake of brevity will not be repeated here. However it should be additionally stated that in shallow water the decay coefficient is not constant and varies with vessel speed and water depth, [63].

Wave Theory

In deep water, when vessel speed is generally less than the critical depth Froude number, the resultant wave pattern is well known and understood. However, in shallow water, the critical and super-critical wave patterns have received less attention.
Depth Froude Number
The dimensionless depth Froude number, \(F_{rh}\), is used to describe the flow regime a vessel is travelling in.

\[
F_{rh} = \frac{V}{\sqrt{gh}} \quad \text{Equation 2}
\]

For any given speed (V), the wave pattern is dependent on water depth (h). Each flow regime has its own characteristic wave pattern. There are no set numerical boundaries between regimes, (sub-critical, critical, super-critical), but rather transitional zones in between identifiable states.

Sub-Critical Wave Pattern
The deep water or sub-critical Kelvin wave pattern, \((\approx F_{rh} < 0.5)\), \([10, 11]\), consists of two main wave systems; transverse and divergent, both systems travelling at the same speed as the vessel. The transverse waves travel parallel to the vessel sailing line. The divergent waves propagate away from the vessel at a constant rate, dissipating their energy into increasing numbers of wavelets. The period of both wave systems are dependent on vessel speed.

Figure 19 (a) illustrates the deep water wave pattern. For clarity only a single wave system is shown, (i.e. a single point source). A complete vessel’s wave system consists of dual wave systems, being the bow and negative stern systems, (i.e. dual interacting point sources). The cusp line of the divergent system intersects the sailing line at a fixed angle of 19°28’.

It should be noted that the effect of water depth on a vessel's resistance may occur at \(F_{rh} \leq 0.5\), \([12]\), with the flow regime varying as early as \(F_{rh} < 0.4\).
Critical Wave Pattern
From theory, the critical wave pattern occurs at $Fr_h = 1.0$. This is when the speed of the point source equals $\sqrt{(g.h)}$, the transverse wavelength becomes infinite so that the whole transverse system is lost and only the divergent system remains, [11].

The angle of this divergent wave is approximately 90 degrees; see Figure 19(b). From the sub-critical condition the divergent wave angle gradually increases to this value. Conversely from the critical condition to the super-critical the wave angle gradually decreases again.

It is known that if a vessel maintains "critical" speed, for any significant length of time, it is unable to maintain a steady state (flow) condition and the periodic generation of waves of translation, or solitons, occurs. The unsteady flow state is also reflected in the vessel's sinkage, trim, and resistance measurements [68, 19].

The blockage coefficient, $(A_w/b.h)$, is the dominant parameter for soliton generation in canals, rivers or towing tanks [19], soliton amplitude and period being functions of water depth, tank width, and depth/draught ratio $(h/T)$, [63,22].

When defining depth Froude number ranges it may be simpler to define the sub-critical and super-critical as steady state regions in which no transients occur in the vessel's sinkage, trim, and resistance measurements. For the general case, the trans-critical depth Froude number range can nominally be defined from $0.8 \leq Fr_h \leq 1.2$, [19, 25].

Super-Critical Wave Pattern
Past the critical speed, the wave pattern consists entirely of a dispersive divergent wave system, the angle of which progressively reduces with speed; see Figure 19 (c).

Water Depth
It is known that water depth has a significant effect on vessel performance and in particular its wave generation and propagation. In general these are due to the increased potential flow around the hull caused by its proximity to the bottom [69] and the effects of depth on wave height and period.

"A body of water is considered to be shallow when the boundaries are close enough to the ship to affect its resistance, speed, attitude, manoeuvring, and other performance characteristics as compared to its corresponding behaviour in a body of water of unlimited depth." [20].
At very low depth Froude numbers, it might be supposed that shallow water effects would be absent, but this is not the case. At low enough water depths, shallow water effects on performance may still be experienced at all depth Froude numbers.

It would seem, therefore, that the effects of water depth cannot be characterized by depth Froude number alone so another parameter is required. It is common to relate water depth to some geometric parameter of the vessel; for displacement craft whose draught is sensibly constant, depth/draught is commonly used, but for high speed craft whose running draught varies while wetted length tends not to, depth/length is preferred. In general terms, therefore, depth Froude number helps define whether the speed is sub-, trans- or super-critical, while depth/length can be said to define the “shallowness” of the water.

**Results**

**Leading Wave Angle**
The bow wave angle, (α), was measured for both hull forms, at varying water depths, across the trans-critical Froude depth range. This was achieved via physical experiments at the AMC MTB; the results of these are plotted in Figure 20.

Across the trans-critical range the bow wave shape changes in form, but can generally be described as cusp like. Due to this transitory nature of form, a simplified measure of the bow wave angle was utilised, being the angle from the probe one to probe five. While not theoretically rigorous, it was considered a good engineering approximation.
Figure 20 – Leading Wave Angle ($\alpha_{lead}$) as a Function of $Fr_h$
The wave angles measured from the physical tests correspond closely with the theory and reported sources, see Table 3.

<table>
<thead>
<tr>
<th>Leading Wave Angle ((\alpha_{\text{lead}}))</th>
<th>Theory</th>
<th>Reported*</th>
<th>Measured**</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sub Critical</strong>( \text{Fr}_h \leq 0.5 )</td>
<td>19°28'</td>
<td>19°28'</td>
<td>20°</td>
</tr>
<tr>
<td><strong>Critical</strong>( \text{Fr}_h =1.0 )</td>
<td>90°00'</td>
<td>78°</td>
<td>80° ***</td>
</tr>
<tr>
<td><strong>Super Critical</strong>( \text{Fr}_h \geq 1.4 )</td>
<td>45°58'</td>
<td>45°30'</td>
<td>43°</td>
</tr>
</tbody>
</table>

* As reported in PNA [27], ** average value, *** \(\alpha_{\text{lead}}\) measured occurs at \(\text{Fr}_h =0.9\)

Table 3 – Leading Angle Comparison

Havelock [7] predicted the divergent leading wave angle to peak at \(\text{Fr}_h = 1.0\). However the measured peak leading wave angle appears to occur at a depth Froude number of about 0.9.

**Critical Depth Froude Number**

It would be expected that the physical test peak values of \(\alpha\) would coincide with the theoretical predictions, that is occurring at the critical depth Froude number. In Table 3 it is seen that measurements indicate that this was not the case, a feature further demonstrated in plots of the variables \(n\) and \(\gamma\) in Figure 17 and Figure 18 which show the peak values occurring at approximately at a depth Froude number of 0.9, less than the expected value of 1.0.

From a review of Havelock’s 1921 seminal paper [70], references are made to this phenomenon. To re-quote Taylor [71], "It was at one time supposed that the speed for a maximum increase in vessel resistance was that of the wave of translation. This, however, holds only for water whose depth is less than 0.2 times the length of the vessel. For greater depths the speed of wave of translation rapidly becomes greater than the speed of maximum increase in resistance"

While Taylor discusses of total vessel resistance, and not wave amplitude, in shallow water it is reasonable to assume that the changing wave pattern accounts for the said changes in resistance. Dand *et al.* [61,68] and the SWIM report [20] references this occurrence with respect to the peak \(C_R\) and \(H_w\) values occurring before unity, at around \(\text{Fr}_h = 0.9\).
Koefoed-Hansen et al. [72], specifically dealing with wash are quoted as, "The critical Froude number is slightly smaller than the theoretical value of one, which is in agreement with experience from towing tank tests and CFD calculations".

There are several reasons for the physical model tests to differ from the theory in this way. Havelock [70] expresses this himself, "the present calculations are based upon a surface pressure of specially simple type, one symmetrical round a point".

The physical actuality is far more complicated than a single pressure source. It can be generalised that non-linear viscosity effects, not accounted for in the theory, account for the differences. Such non-linear effects typically exhibit an “unsteady” or oscillatory type of behaviour, and are functions of water depth, run time and model acceleration, [73, 74, 75, 76].

**Wave Growth**

Solitons are cyclical (time dependant) in nature and their characteristics are dependent on physical parameters relating to the basin and model such water depth, tank breadth, model volume, etc. [19]. The time cycle of a soliton can be approximately described as follows.

At the critical depth number, the vessel’s leading wave will grow from a minimum value, (time = 0%), to a maximum value at its fully developed state (time = 100%). The bow wave (soliton) will then shed forward from the vessel, and a new bow wave will be formed, steadily growing once again to a maximum value. Associated changes in sinkage and trim follow this wave growth cycle, with the trim reaching a maximum at time = 100%. This oscillatory behaviour will continue as long as time permits.

From the test results soliton generation has been recorded within the wave cuts and observed from photographs. However as the basin has a finite length, it has been unable to record a complete soliton cycle. It follows that the wave amplitudes measured at the critical number and the subsequent decay estimated, are “snapshots” of this time dependant phenomena. Whittaker [20] in the SWIM report realised similar limitations of working in a relatively short model test basin.

**Future Work**

To investigate further the shallow water phenomenon discussed, it is proposed to supplement the existing model test programmes with a) additional lower depth Froude number runs and b) additional deeper h/L conditions. It is supposed that a deep water h/L ratio surface limit will be established from these tests.
If so, a combination of parameters representing vessel speed / length / water depth may be required to fully delineate deep water from shallow water and the transitional zone in between, (i.e. $F_{rh}$, $h/L$, $F_{rL}$).

**Concluding Remarks**

Two hull forms were tested in shallow water covering the trans-critical depth Froude number range focussing on the leading wave angle. The observed wave patterns were in accordance with theoretical predictions for the sub-critical, critical and super-critical ($F_{rh}$) operational zones.

The leading wave angles for these zones were measured and it was shown that the form of the leading wave angle curve, (as a function of $F_{rh}$), closely corresponds to that predicted by theory, albeit with the peak value occurring closer to $F_{rh} = 0.9$ than $1.0$. It is probable that these differences in peak values were due to the non-linear viscosity effects present in the physical tests and not accounted for in the numerical theory. Importantly the relation is independent of hull form or depth – draught ratio, and so the leading wave angle can be utilised as a shallow water wash characterisation.
5. **Wash Unsteadiness**

This research was originally published as;

**“Subcritical Wave Wake Unsteadiness”**

Alex Robbins¹, Giles Thomas¹, Martin Renilson¹, Gregor Macfarlane¹, Ian Dand²

¹ Australian Maritime College, Tasmania
² BMT Isis, Fareham, UK


For the avoidance of repetition the original paper has been modified for this thesis.
Paper Abstract
Vessel wash in deep water is well understood, shallow water less so, specifically the effect of restricted water. This operational zone is highly dynamic and non-linear in nature, thus being worthy of closer examination. The paper reviews the primary mechanisms for unsteadiness in wash: starting acceleration and soliton generation. A comprehensive set of experiments was conducted using an NPL catamaran hull form to investigate unsteadiness in both wave height and wave angle. The results show that the unsteadiness was primarily due to soliton generation, and that blockage has a significant effect. As a result, additional characterisations, aimed at defining shallow water effects in the trans-critical region, are proposed.

Introduction
As part of an ongoing research programme investigating wash in shallow water, multiple physical tests have been completed at AMC MTB.

As prefaced in Chapter 3, a review of the wave height decay results for trans-critical wash revealed some unexpected findings, which were possibly due to “unsteadiness”. The term “unsteadiness” relates to the variation of a vessel’s (measured) wave pattern over time. One type of unsteadiness is connected with the way the wave system is initiated within the towing tank or model basin (transient state). The other type is connected to trans-critical effects (steady state) where the critical condition is defined as that existing when depth Froude number is unity.

These findings therefore warranted further investigations into; (i) starting accelerations and (ii) soliton formation. This resulted in an additional test programme, on which this chapter is based.

Understanding and separating the two categories of unsteadiness is important when analysing experimental and actual vessel wash. Without it incorrect conclusions may be drawn from the analysis.

Unsteadiness Due To Acceleration
As already mentioned, one type of unsteadiness is connected with the way the wave system is initiated within the towing tank or model basin. This form of unsteadiness is a function of model starting acceleration and running velocity. It can be described as an oscillatory transient decay phenomenon.
Havelock

In his papers of 1948 and 1949, Havelock [73,74] described calculations for the wave resistance of a submerged cylinder, in particular the effect on wave resistance when the cylinder is accelerated from rest to a constant velocity.

His analysis determined that the wave resistance oscillated about a steady mean value for a steady state speed, with the oscillations reducing over time. The key question he posed was - “how long is it before the effect of the starting conditions becomes inappreciable?” [73].

Havelock determined that the higher the starting acceleration, the lower the oscillation peak value and the quicker the oscillations decay to a steady mean value. In effect the starting acceleration “ramp” has a direct effect on wave pattern resistance and in turn, one can assume, the wave pattern itself.

It must be noted that Havelock's findings related to the calculated wave pattern resistance of a point source – an entirely mathematical postulate. Also these were calculations done for deep, not shallow water. Unlike Havelock, this study is not using calculated wave pattern resistance, but using the actual wave elevation measured in model tests.

Wehausen

Wehausen expanded on Havelock’s work in his 1964 paper [75], providing more realistic results by utilising thin ship theory rather than point sources. He confirmed Havelock’s conclusions that the temporal oscillations in measured resistance were due to the initial acceleration.

Wehausen noted that the form of oscillation decay is asymptotic, which suggests that the oscillations will never entirely decay away. The key question, (as earlier posed by Havelock), is when does this effect become insignificant? In his paper Wehausen stated; “No matter how quickly the final desired speed is attained by the model, there always remains some question as to how long the influence of the initial accelerations persists and what form this takes” [75].

Wehausen provided more numerical examples than Havelock and showed too that calculated wave resistance oscillated about the mean as a function of starting acceleration.

Towing tanks had long reported oscillations in resistance measurements, which were primarily thought to come from mechanical resonance within the resistance dynamometers utilised. Experimenters had taken a mean line though the oscillations within the resistance record, and Wehausen’s paper confirmed this method as correct.
An interesting observation was made by Wehausen et al. [77] during some shallow water experiments. Tests run in the Berkeley towing tank examining the effect of “bottom irregularities” on Series 60 models, produced an unexpected outcome. A slowly decaying, periodic response was observed within the wave trace, when the model was driven over the bottom irregularity (a submerged block), or when the model was “hard started” from rest. Wehausen termed this phenomenon as “ringing”.

Wehausen noted that variation of the bottom irregularity had little effect on the ringing phenomenon whereas water depth, model speed, and tank breadth did. Wehausen concluded that this phenomenon is the result of multiple wave reflection from the tank boundaries and has no practical significance for ships. However it should be noted that ships have been known to begin pitching due to bottom irregularities [78].

**Calisal**

Calisal in his 1977 paper [76] extended the initial acceleration work of Havelock and Wehausen by reviewing the theory with respect to wave pattern measurements. At this time the direct determination of a vessel’s wave pattern resistance from wave probes was in vogue. Calisal determined that: “the Fourier transform of the wave-height record, provides a possibility of detecting the existence of the initial acceleration effects in the wave system” [76].

Calisal found that wave pattern resistance deduced from longitudinal cuts was unaffected by initial acceleration (although the wave spectrum might be), whereas that deduced from transverse cuts was. “One can claim, therefore, that the wave resistance calculations based on fixed probes and longitudinal cuts are not affected by initial acceleration, even though the wave spectrum is.” [76].

**Doctors**

The previous papers of Havelock, Wehausen and Calisal cover the effects of starting acceleration in deep water, and do so by numerical calculation only. Doctors [21, 79] moved the analysis into shallow water and in particular the trans-critical regime, utilising numerical and experimental results.

Doctors’ calculations showed that peak wave height values shift with longitudinal probe location. Furthermore, he concluded that these effects can be largely attributed to the transverse component of the wave system. The question remains how applicable is this finding to the critical condition?
Unsteadiness Due To Solitons

As prefaced, one type of unsteadiness is due to trans-critical effects: the generation of solitons. This gives rise to another form of unsteadiness which is a function of water depth, displacement, and side boundary; it is a periodic/oscillatory phenomenon even under steady state conditions.

In 1834, John Scott Russell observed the “wave of translation” (i.e. soliton) in a Scottish canal. His discovery, later detailed in a paper [18], was not fully exploited until digital computers were able to demonstrate its application in signal transmission. His work is also of significance for vessels operating in shallow water. The behaviour of solitons is the subject of multiple theoretical and physical studies, for example [80, 81, 82].

By definition a soliton is a single wave with no preceding (or following) trough. In general solitons are cyclical and therefore time dependant. The time step cycle of a soliton can be approximately described diagrammatically for a vessel travelling at constant speed in Figure 21.

![Illustration of Soliton Life Cycle](image)

Figure 21 - Illustration of Soliton Life Cycle

Travelling at the critical depth Froude number, the vessel’s leading wave will grow from a minimum value, (time = 0% - point A), to a maximum at its fully developed state (time = 100% - Point D). The leading wave (soliton) will then shed forward from the vessel (Point E), and a new leading wave will be formed, steadily growing once again to a maximum value.
While the model speed remains constant, there are associated changes in resistance, sinkage and trim which follow this wave growth cycle. This oscillatory behaviour will continue as long as time (i.e. facility length) permits.

Ertekin et al. note that an important property of a soliton is that its speed is supercritical [19], so once the wave is generated and then shed from a model, it will travel along the tank, leaving the model behind, for the cycle to restart afresh.

It follows that for a vessel travelling at, or around the critical depth Froude number, (where soliton growth is present), any wash measurement made will be time-dependant. Figure 22 is a conceptual diagram of this time dependency. The wave elevation at “A” is significantly different to that at “E”, where a full soliton has been formed and shed. Specific tests in a larger facility would be required to fully populate such a diagram, and determine its actual shape with respect to magnitude and form.

![Figure 22 - Conceptual Diagram of Time Dependant Wave Height](image)

**Restricted Water**

The flow around a vessel can be restricted vertically (i.e. depth), or laterally (i.e. width), or both vertically and laterally giving rise to the effect known as blockage [83].

To quote Saunders, [42], "A body of water is considered to be shallow when the boundaries are close enough to the ship to affect its resistance, speed, attitude, manoeuvring, and other
performance characteristics as compared to its corresponding behaviour in a body of water of unlimited depth”. In general this is due to the increased potential flow around the hull caused by its proximity to the bottom [69].

The most common water restriction is depth, which is generically termed as “shallow water”. In general terms the depth Froude number helps define whether the speed is sub-critical, critical or super-critical, while depth-length (h/L) or depth-draught (h/T) ratio may define the “shallowness” of the water in an absolute sense.

It is not common to have the flow around a vessel restricted laterally only. More commonly both lateral and vertical flow restriction occurs, such as when a vessel travels in a canal or channel and experiences blockage. An experimental test facility, either towing tank or model basin, will restrict the flow around a model.

Blockage can be described numerically in terms of the blockage coefficient (k), which is a function of vessel cross sectional area (Aₜ), tank width (b), water depth (h) in the form k = Aₜ / (b.h).

The effect of blockage on vessel speed loss, (or increased resistance), has been well covered in the literature: Schlichting [69], Lackenby [84], Baker [85] and Landweber [86]. However the blockage effect on vessel wash has received less attention.

One core concern is that a towing tank does not have the ability to vary breadth in order to explore the effect of lateral restriction, while the broader model basin with a much reduced width restriction does not necessarily have the length required to see a full soliton cycle.

**Soliton Generation**

Lamb [11] has demonstrated that solitons occur at the critical depth Froude number, which does not account for the occurrence of solitons occurring at higher, (or lower), speeds than at the critical depth Froude number observed in this study. Remembering that depth Froude number is a simplistic, depth specific measure, based on a single travelling pressure source, it is known [63, 22], that soliton amplitude and period are functions of water depth, tank width, and depth/draught ratio (i.e. blockage).

Ertekin [19] conducted experiments to investigate the effect of blockage coefficient variation on soliton generation. He observed solitons occurring over a range of depth Froude numbers from Frₜ 0.9 to Frₜ 1.2, not just at the critical value. It was further reported that, for both measured and calculated experiments, the resistance oscillated about a mean value, with a period equal to that of soliton generation.
Some of his key findings were that: (a) for an increase in tank width - soliton wave amplitude decreases; (b) for an increase in tank width - soliton period increases; (c) the soliton amplitude curve characteristics are almost identical for the same blockage value, despite significant change in water depth.

Lyakhovitsky [87], in his numerical investigations of ships travelling in a canal, presented the idea that rather than being a single fixed point at $Fr_h = 1$, there are separate boundaries between sub-critical, critical and super-critical flow states. Furthermore, that these critical boundaries diverge with increasing blockage.

Figure 23 is a direct reproduction from Lyakhovitsky’s book, and describes the sub-critical, critical and super-critical boundaries. Said boundaries are obtained from his novel one-dimensional hydraulic theory, which while not theoretically rigorous, are in accord with the present results.

The depth Froude number used in combination with the blockage coefficient may therefore provide clearer definition of the flow regime a vessel is travelling in, according to its physical surroundings.
Current Investigation

As aforementioned, results from the shallow water wash decay investigation in Chapter 3 highlighted some unexpected results related to unsteadiness. From the theory above it is clear that said previous experiments may have experienced shallow water unsteadiness, probably related to blockage and depth Froude number. Therefore an additional test programme was proposed to investigate the possible causes of this unsteadiness.

The key questions are;

a) Has the wave pattern settled to a steady state by the primary measurement location?

b) What is the limit of the unsteadiness?

c) How does this affect the experimental results?

Physical Tests

Two hull forms were tested, one a low wash type, the other a conventional fuller form. The tests were carried out in shallow water at the AMC MTB. The model test basin, the test programme, measurement and data acquisition, and hull forms utilised have been fully described in Chapter 2, and for the sake of brevity will not be repeated here.

Experimental Results

Leading Wave Characterisation

As covered in Chapter 4, the use of the maximum wave height ($H_w$) as the primary measure across the trans-critical range was not recommended.

Wave cut analysis of the measured data showed significant superposition of competing wave systems. In addition the position of the maximum wave height changed unexpectedly across the transverse probe array, providing erroneous results.

Instead the bow wave’s height ($H_{bow}$) was utilised as a suitable characterisation because it is easily identifiable across the trans-critical range and still provides similar decay characteristics. It has since been recognised that the use of the term “bow wave”, in the far field, may not be entirely accurate. Therefore, a more suitable alternative, “leading wave” ($H_{lead}$) been utilised as the primary wave height measure within this work.

Error Analysis

Error analysis, (uncertainty analysis), is important in establishing a baseline of confidence for any data set. For this work it has been not only utilised to establish accuracy, but also in determining if wave growth is occurring. The standard deviation method was utilised to
establish a baseline error for these tests, (where standard error = standard deviation / √ number of observations).

For this, repeat runs were made at a fixed depth across the sub-critical range, and also at a fixed depth Froude number across multiple water depths, (H_{lead} was utilised).

Repeat runs were completed across the subcritical range, (Fr_{h} = 0.5 - 1.0), at a fixed water depth of 200mm. Figure 24 is a plot of the H_{lead} Standard deviation, as a function of Fr_{h}, for a number of different transverse offsets. From this plot the maximum standard deviation across all runs is seen to be 0.25.

![Figure 24 - Error for Fixed Depth (200mm)](image)

Repeat runs were completed at the four water depths (h = 200, 400, 600, 800 mm) at a fixed Fr_{h} of 0.5. Figure 25 is a plot of the H_{lead} standard deviation for each water depth, and a number of different transverse offsets. From this plot the maximum standard deviation across all runs is again 0.25.
For constant \( Fr_h \) and constant water depth – across all transverse probes – the error appears to be relatively constant with a maximum value of 0.25, which is in keeping with Chapters 3 and 4. It was therefore assumed that any standard deviation value above 0.25 would be an indicator of growth or unsteadiness.

### Wave Growth

To determine if a vessel’s wash is unsteady the main longitudinal array of wave probes was utilised, refer Figure 6. A comparison of the resultant measured wave heights at various longitudinal locations revealed if growth was occurring.

The results are as plotted in Figure 26 and it is clear that there is significant leading wave growth occurring around the critical \( Fr_h \) number, whereas at low \( Fr_h \) numbers there is no growth detected.

Standard deviation shows how much variation there is from the average or mean, but does not indicate growth (positive value) or decay (negative value). It was decided that a more descriptive measure was required, which would indicate if growth or decay (or neither) was occurring.

Accordingly, the \( dy/dx \) value of a line fitted through the (averaged) points of longitudinal \( H_{lead} \), plotted against distance run, was utilised. From error analysis the \( dy/dx \) baseline error was determined to be 0.1. The results are in Figure 27, which displays very similar results as for Figure 26.
The lowest water depth (200mm) exhibits the highest level of growth, (both standard deviation and slope), with growth detected from $F_{Rh} 0.80$. The highest water depth (800mm) exhibits the lowest level of growth, (both standard deviation and slope), with growth detected from $F_{Rh} 0.65$.

The light condition, ($L/\sqrt[3]{V} = 8.5$), results exhibit marginally lower levels of growth than those for the heavy condition, ($L/\sqrt[3]{V} = 9.2$), albeit at a higher $F_{Rh}$ value. Note that $T$ and $k$ are both greater for the heavier displacement, with $h/T$ being smaller.

From these results it can be concluded that wave growth is occurring within the experiments, and occurs at speeds as low as $F_{Rh} = 0.7$. Water depth has a significant effect on wave growth with respect to its level and inception point, while displacement has only a secondary effect. These findings have been summarised in Table 4.
One possible cause of the unexpected results, as prefaced in Chapter 3 was that the wave system was not fully developed. To investigate this claim, the leading wave angle was measured at multiple positions longitudinally utilising the transverse wave probe arrays.

Referencing Figure 6 the leading wave angle was measured using probes 1 + 9 on the main transverse array, probes 12 + 13, and probes 10 + 11 on the secondary transverse arrays. Each array was spaced longitudinally at 2m, (0.8L).

The leading wave angle was measured from Frh 0.3 – 1.0, at 200mm, 400mm, 600mm, and 800mm water depths. The data was processed to provide the leading wave angle from each run at each array.

The results are in keeping with the earlier leading wave angle findings, (Chapter 4), namely that the peak value occurs around Frh = 0.9, (not at Frh = 1.0) and the wave angle remains at the “deep water” value of 19°28’ for Frh < 0.7. These findings have also been summarised in Table 4.

Figure 28 shows a plot of leading wave angle as a function of depth Froude number, measured at the three arrays, for h = 800mm. It is clear that for Frh < 0.7 there is little or no

Figure 27 - Leading Wave dy/dx
variation in measured angle between the arrays. This is a trend which continued for the other three water depths tested.

For $Fr_h > 0.7$ there is a slight divergence in the measured angles, although this is within the error margin. Similar trends are found for the other depth conditions, although not as pronounced. The results indicate that the leading wave angle has settled, across the depth Froude number range, at all water depths tested.

Figure 28 - Leading Wave Angle ($h = 800$mm)
5 Wash Unsteadiness

<table>
<thead>
<tr>
<th>Condition</th>
<th>Depth Froude Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
</tr>
<tr>
<td><strong>C 8</strong> h = 200mm k = 0.0141</td>
<td>Steadiness 1</td>
</tr>
<tr>
<td>Bow Wave Angle 2</td>
<td>Deep</td>
</tr>
<tr>
<td>Soliton 3</td>
<td>None</td>
</tr>
<tr>
<td><strong>Zone 4</strong></td>
<td>1</td>
</tr>
<tr>
<td><strong>C 5</strong> h = 400mm k = 0.0070</td>
<td>Steadiness 1</td>
</tr>
<tr>
<td>Bow Wave Angle 2</td>
<td>Deep</td>
</tr>
<tr>
<td>Soliton 3</td>
<td>None</td>
</tr>
<tr>
<td><strong>Zone 4</strong></td>
<td>1</td>
</tr>
<tr>
<td><strong>C 4</strong> h = 600mm k = 0.0047</td>
<td>Steadiness 1</td>
</tr>
<tr>
<td>Bow Wave Angle 2</td>
<td>Deep</td>
</tr>
<tr>
<td>Soliton 3</td>
<td>None</td>
</tr>
<tr>
<td><strong>Zone 4</strong></td>
<td>1</td>
</tr>
<tr>
<td><strong>C 1</strong> h = 800mm k = 0.0035</td>
<td>Steadiness 1</td>
</tr>
<tr>
<td>Bow Wave Angle 2</td>
<td>Deep</td>
</tr>
<tr>
<td>Soliton 3</td>
<td>None</td>
</tr>
<tr>
<td><strong>Zone 4</strong></td>
<td>1</td>
</tr>
</tbody>
</table>

Steadiness 1 = dy/dx < 0.1 (as per Figure 27); Bow Wave Angle 2 = $\theta_{bow} < 20$; Shallow = $\theta_{bow} > 20$; Soliton 3 = Soliton observed in longitudinal wave cut (@ probe 17); None* = No Solitons observed (possibly due to limited run time); Zone 4 = As specified in Table 5

Table 4 – AMC MTB Results

57
Soliton Generation
As mentioned before, due to the blockage characteristics of the model basin, and taking into account previous experimental studies [19, 22, 63, 68], it was probable that any soliton generated would be of low amplitude and long period.

It was decided that soliton identification would best be achieved by inspection of longitudinal wave cuts and further supported with enhanced photographs.

In addition to the physical wave probe measurements, many photographs were taken of the experiments. To enhance the standard photo an array of white rope lights were fixed to the basin’s ceiling. The lights’ reflections on the water’s surface made identification of the resultant wave pattern clearer. While not a true measurement, these images provided an excellent identification aid, see Figure 29.

The longitudinal cut utilised for inspection was taken from Probe 17 on the main transverse array. This probe was the most outboard and forward of the probes available, being clear of possible local wave effects, and allowing maximum wave development.

Figure 29 - Fr_h = 1.0, h = 200mm – Soliton

From observations of the enhanced photographs and longitudinal wave cuts, a summary table, (Table 4), has been generated. This table highlights when a soliton has been observed
in the photograph and in the wave cut. The wave cut for the observed soliton has been reproduced in Figure 30.

From a closer review of the wave cuts, for example Figure 31, \( i.e. \) run 92, \( Fr_h = 0.9, h = 200\text{mm} \), it can be seen that they exhibit a similar wave profile to that of Figure 30, albeit under-developed. Furthermore these wave cuts, showing an under-developed soliton form, closely match those runs also exhibiting unsteadiness, \( \text{ref Figure 26} \).

This suggests that there is the possible onset of a soliton. However due to limited run time, \( i.e. \) limited facility length, it has not developed sufficiently to be identified correctly as a soliton. This has been noted in Table 4 as “Onset”.

While only one soliton was clearly observed, it is possible that given more run time, solitons would also have been observed at the deeper water depths of \( h = 400, 600, \text{and} 800\text{mm} \), \( \text{Ref Figure 32, Figure 33 and Figure 34} \). Accordingly these runs have been noted in Table 4 as “None”. Furthermore it is proposed that for some near-critical runs at shallower water depths, given more run time, solitons would have developed.

Figure 35 is a plot of results from longitudinal probes for \( Fr_h = 1.0 \) and \( h = 200\text{mm} \). This figure clearly shows the development of the soliton. A review of probe 17 for \( Fr_h = 0.9 \) shows a similar wave form as those appearing at 14 for \( Fr_h = 1.0 \). This would suggest that, given more run time, fully developed solitons could occur at the lower depth Froude numbers.
Figure 30 - Frh = 1.0, h = 200mm - Soliton

Figure 31 - Frh = 0.9, h = 200mm - Soliton Onset

Figure 32 - Frh = 1.0, h = 400mm – No Soliton Detected

Figure 33 - Frh = 1.0, h = 600mm – No Soliton Detected

Figure 34 - Frh = 1.0, h = 800mm – No Soliton Detected
Figure 35 - Fr_h = 1.0, h = 200mm, Heavy – Soliton Development
A further review of Figure 30 shows a smaller peak forming directly behind the soliton, which suggests that a second soliton may be forming. This is in keeping with the oscillatory nature of solitons, as seen in other experimental works [19, 22].

Dand et al. [22], in their work on catamarans operating in shallow water, reported; “Rather than a single wave, the disturbance ahead of the model consisted of a number of solitary waves with the process of creation occurring continuously along the tank. This suggests that the assumption that such waves are created simply by the starting transients of the models at the beginning of a run is incorrect.” Therefore the presence of a second soliton would indicate that the observed unsteadiness is not caused by starting accelerations.

Figure 36 is a composite of Figure 23, (Lyakhovitsky plot), and Table 4, (results summary table). Results from the NPL+ hull form have been added to the original diagram. The blockage ratios for each of the four water depths investigated are indicated in this figure.

There is some agreement with the numerical predictions, although greater concurrence may have been achieved had model run time been longer, (enabling full soliton development near the critical Frh). Additionally an increased number of runs, at finer Frh resolution, might have revealed a clearer definition of the behaviour.
Figure 36 - Critical Boundaries with Measured Data Points
Discussion

Unsteadiness
As already discussed, unsteadiness in the wave pattern can be accounted for by either starting accelerations, or from the generation of solitons near the critical depth Froude number. Starting acceleration unsteadiness can be discounted primarily as there was no evidence of it at low speeds. Secondary soliton generation at higher speeds confirms this finding.

However despite the runs at Frh 0.7 and 0.8 showing unsteadiness, (Figure 27), and shallow water leading wave angle, (Figure 28), no solitons were observed within the longitudinal wave cuts. This finding concurs with previous shallow water testing at the AMC MTB, where no solitons were observed at such low Frh.

The question remains - what is causing the recorded unsteadiness? It is possible that these sub-critical, shallow water, leading waves are still developing, and a longer model run time may show them develop to a steady state, (but not into solitons). It is equally possible that these runs are close to the transitional boundary between the sub-critical and critical zone.

While only one soliton was actually recorded, (at the shallowest water depth and at the critical number), the onset of solitons was observed at the near critical Frh of 0.9, (Figure 31). This suggests that the Lyakhovitsky theorem [87], (of blockage dependant boundaries between sub-critical, critical and super-critical flow states), is correct.

Operational Zones
For this work the definable wash zones are: (1) Sub-Critical Deep, (2) Sub-Critical Shallow and (3) Critical Shallow. The characterisations (or metrics) used to define each zone are; (a) Leading Wave Angle, (b) Steadiness and (c) Soliton. The criteria for each zone, (i.e. proposed shallow water metrics), are shown in Table 5.

The metrics given in Table 5 relate to those parameters which indicate whether or not the vessel is experiencing different physical conditions as a result of the proximity of the bottom. For example, if the leading wave angle is different to that in deep water, then this is an indicator that the vessel is influenced by the bottom in this case. Equally, if the wave height is varying with time (or distance along the x axis) then this indicates that physically the vessel is in a regime other than ‘deep’ water.
5 Wash Unsteadiness

<table>
<thead>
<tr>
<th>Zone</th>
<th>Metric</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Steadiness 1</td>
</tr>
<tr>
<td>1 Sub-Critical Deep</td>
<td>dy/dx &lt; 0.1</td>
</tr>
<tr>
<td>2 Sub-Critical Shallow</td>
<td>dy/dx &gt; 0.1</td>
</tr>
<tr>
<td>3 Critical Shallow</td>
<td>dy/dx &gt; 0.1</td>
</tr>
</tbody>
</table>

Steadiness 1 = dy/dx < 0.1 (as per Figure 27)
Bow Wave Angle 2 = Deep = \( \theta_{bow} < 20^\circ \); Shallow = \( \theta_{bow} > 20^\circ \)
Soliton 3 = Soliton observed in longitudinal wave cut (@ probe 17)

Table 5 – Proposed Shallow Water Metrics

Furthermore, said metrics may now be used to determine whether the vessel, at the given speed, water depth, draught, and blockage is influenced by the presence of the bottom. Parameters commonly used such as \( h/T \) and \( F_r \) do not do this, but are simply non-dimensional values which indicate how close the vessel is to the bottom, or at what speed it is travelling. On their own they are *not* measures of whether a particular vessel in a particular condition is influenced by the presence of the bottom, or not.

Table 4 summarises the test results, utilising the aforementioned metrics and resultant zones, for each \( F_r \) and water depth. These outcomes have also been included into Figure 36.

Critical Wash Measurement

From the test results, soliton generation has been recorded within the wave cuts and observed from photographs. However as the basin has a finite length, it was not possible to record a complete soliton cycle. It follows that the wave patterns measured around the critical number are “snapshots” of this time dependant phenomenon. Furthermore any measurement needs to be referenced by a “time stamp” of where it occurs within the cycle, reference Figure 21.

There exist high and low \( F_r \) limits outside which wave growth can be considered as negligible, and the wave pattern could be considered as steady. These zones could be known as “shallow water steady” and “deep water steady”. The zone inside these limits could be known as “shallow water unsteady” zone, as shown in Figure 23 as the critical zone.
Effect on Wave Decay
In Chapter 3, covering the trans-critical zone (Frh 0.3 - 2.5), the results of leading wave decay were presented.

The results showed a large variation in the decay coefficient (n), specifically around the critical number. It is now clear that these measurements were most likely unsteady and time dependant. In turn the decay coefficient presented was affected and it is worth reiterating that any measurements taken within this unsteady zone are a snap shot within the cycle, and should be qualified by a time stamp.

Variation of Displacement
It was shown in Chapter 3 that, while displacement has a significant effect on wave height, it appears to have an insignificant effect on wave growth, (i.e. unsteadiness). This may be because of the relatively low blockage (k = 0.0035) at which the effects of displacement were tested.
It has been suggested that blockage seems to be the key factor in unsteadiness [87]. It is probable that at higher blockages, displacement will have an increasingly significant effect. Further testing would be required to determine this.

Facility Limitation
It is known that soliton generation is a function of physical parameters such as tank width, water depth, model speed, model acceleration, model displacement and tank length. Accordingly wave measurements should be considered specific to the facility and the models tested within it.

These tests have shown that boundaries exist for unsteadiness within the AMC MTB. These boundaries are functions of blockage, water depth and model speed. It is probable that model hull form and hull arrangement (i.e. monohull / catamaran), also have an effect on said boundaries. It is recommended that more tests be undertaken to develop a comprehensive understanding of shallow water wave growth within the AMC MTB. This would require large permutations of water depth, facility width, model speed, model hull form, model draught, model arrangement.

Determining the complete global understanding of shallow water wave growth would require significant work. As such, near critical unsteadiness, (i.e. wave growth), should be considered a standalone topic.
Concluding Remarks

Physical testing, investigating shallow water wash decay and discussed above, highlighted some unexpected findings. In turn, further physical tests into the effect of starting accelerations and soliton formation were completed.

These tests determined that, as the critical number is approached, it is most likely that the formation of solitons, not starting accelerations, which are the cause of the reported unsteadiness. For the lower Frh runs which exhibited unsteadiness, it is most likely that insufficient run time is highlighting an under-developed leading wave.

Furthermore, the blockage of the test facility is a key consideration in sub-critical wash, having a potentially significant effect on the measured results. It is recommended that depth Froude number and blockage should be utilised as a combined measure of shallow water unsteadiness.

It is noted that due to the time-dependant nature of near-critical unsteadiness, wash measurements should be referenced with a time stamp. Also, that more research is required to understand this problem fully.

The wider (i.e. full scale) implications of these model test findings are potentially significant. All previous wash measurements, taken around the critical Frh, will be time-dependant (i.e. unsteady). This is especially true in high blockage environments such as rivers or canals. A new metric may be required to measure this unsteadiness adequately.
6. **Wavelet Analysis Characterisation**

This research was originally published as;

**“Vessel Wave Wake Characterisation Using Wavelet Analysis”**

Alex Robbins¹, Giles Thomas¹, Walid Amin¹, Gregor Macfarlane¹, Martin Renilson¹⁺², Ian Dand³

¹Australian Maritime College, University of Tasmania,
²Higher Colleges of Technology, UAE,
³BMT Isis, Fareham, UK


For the avoidance of repetition the original paper has been modified for this thesis.
Paper Abstract
This work focuses on characterising vessel wash (wash) using wavelet analysis when a vessel is operating in the sub-critical and critical zone. Such characterisation complements other wash characteristics: Froude depth number, bow wave angle, solitons and decay coefficient. The examination of experimental results indicates that differences in characteristics with respect to water depth, Froude depth number, vessel displacement, hull form and soliton generation can be identified through wavelet analysis. The results demonstrate “proof of concept” that wavelet analysis is a powerful tool for characterising vessel wash and captures the effects of key operational and vessel changes.

Problem Introduction
Wash Impact
As a wave enters shallow water the dynamics of the wave change; wave speed and wave length decrease and, (following the law of conservation of energy), the wave height, (i.e. steepness), increases. Within wave mechanics this process is known as “shoaling” [88].

The typical wave spectra encountered by a shoreline are those from wind waves, sea swell, and the tide. These spectra are naturally occurring and can essentially be considered as continuous in form; (the tide generally comprising a number of regular sinusoidal waves, albeit of extremely long period). As the shoreline (or river bank) has been formed by long-term exposure to the wave swell and/or tide spectra, the introduction of vessel-generated “foreign” transient wave spectra, can have significant impact on the shoreline and beach-goers [9].

These foreign vessel waves may have a wave height and period differing significantly from those of which occur naturally. In turn this may lead to greater shoaling wave surge or breaking, both of which can endanger shore goers, facilitate shore erosion (redistribution of sand, sediment or rocks) and long term potentially change the local coastal morphological processes [89]. The level of erosion and danger to shore goers is dependent on the form of the shoreline.

Marine traffic in coastal or inland waterways has two immediate effects on the environment: bank erosion and fine sediment re-suspension. This environmental impact has led to “green” criteria being placed on vessel design and operation [25, 26]. Environmental impact is no longer seen as of secondary importance, but in some cases has become the prime concern and its assessment a contractual requirement [27, 28].

Not only can vessel wash affect the environment, but it can also have an effect on other waterway users, (e.g. commercial and recreational vessels, marine infrastructure, swimmers
and shore-goers). It can be a minor irritant affecting the enjoyment of others within the littoral zone; however it can also be a significant public safety hazard.

There has been one recorded death which can be directly attributed to vessel wash; the Purdy Incident [6]. A 10m long monohull fishing vessel, *Purdy*, was swamped by high speed craft (HSC) wash. A crew member was washed overboard and subsequently lost at sea. This was despite the weather being fine and clear, the vessel being headed into the oncoming wash and speed restrictions being placed on the HSC. The official accident report suggested that a solitary may have been to blame, which passed across shoaling water, thereby increasing in size as it approached the Purdy. This incident highlighted the possible extreme danger and unpredictability of wash.

Wash can be identified as a form of pollution and considered in the same vein as noise pollution, (both typically being transient wave forms). Its effects are very much dependant on amplitude, frequency, (i.e. energy) and exposure time [37]. While there is common understanding and awareness of noise pollution, and its effects, there is little for vessel wash. This may be because its impact is often only indirectly experienced by humans.

The aforementioned environmental, safety, and pollution risks have led authorities to implement wash mitigation strategies. Typically these have taken the form of vessel route planning, speed restrictions, access restrictions and shore warnings [26, 50]. Clearly there is a need to understand vessel wash parameters (i.e. characteristics). Ideally such understanding will lead to more focussed mitigating strategies.

**Wash Characterisation**

For any given speed (V), the wave pattern is dependent on water depth (h). Each flow regime has its own characteristic wave pattern. There are no set numerical boundaries between regimes, (sub-critical, critical, super-critical), but rather transitional zones in between identifiable states. A suitable characterisation must fully capture this dynamic.

Macfarlane and Renilson [52] stated that a suitable (deep water) wash measure should be; (i) independent of location, (ii) independent of sample size, (iii) easy to understand, (iv) easy to measure, and (v) able to rank vessel's wake performance.

Current wash characterisations are a combination of observation and calculation. Said characterisations are focussed primarily on the deep water condition. Therefore these measures are unlikely to be effective in assessing shallow water wash. What is required is a wash metric that is applicable to both the deep and shallow water conditions, (i.e. transcritical). Following on from Macfarlane and Renilson's guidelines, such a metric should provide additional information as to the energy (form and magnitude) contained within the
wave system. Wash can be characterised utilising a combination of observation and calculation methods.

Maximum wave height, \( (H_w) \) is the simplest and most obvious wash characteristic, (being the largest elevation from peak to trough within an individual wave). Wave period, \((T_w)\), is another simple wave characteristic, (being peak-to-peak time of \(H_w\)). The \(T_w\) of individual waves or groups of waves is important, especially when coupled with \(H_w\). While good initial indicators of vessel wash, \(H_w\) and \(T_w\) do not fulfil the requirements for a suitable measure of wash, specifically the requirement for a more global understanding of the wash event. It follows that more descriptive measures are needed.

Wave angle is another characterisation primarily based on observation. The angle of the leading wave \((\alpha_{lead})\) has been proposed as a key wash characterisation in Chapter 4 as opposed to the angle of the maximum wave \((\alpha_{max})\). This is because height of the leading wave \(H_{lead}\) is easily identifiable across the trans-critical range and still provides similar decay characteristics to the \(H_{max}\) utilised in sub-critical (deep water) work.

A more complex, and commonly used, wash characterisation is the Froude depth number, \((Fr_h)\), equation 3. Typically utilised to classify the flow regime a vessel is travelling in and hence its wash.

\[
Fr_h = \frac{V}{\sqrt{(g.h)}} \quad \text{Equation 3}
\]

Four recognised transitional zones between identifiable states are known for the Froude depth number; (i) Deep water \((Fr_h < 0.5)\); (ii) Sub-critical \((0.5 < Fr_h < 1.0)\), (iii) Critical \((Fr_h = 1.0)\); and (iv) Super-critical \((Fr_h > 1.0)\). It is understood that the trans-critical Froude depth number range is nominally \(0.5 < Fr_h < 1.5\).

These zones are based on Havelock’s 2D point theory [70], and each corresponds to a Froude depth number range. The physical actuality is of course far more complicated than a single pressure source, and accordingly the actual \(Fr_h\) differ from those theoretically proposed.

Wave energy \((E)\), Equation 4, is a measure of the energy contained in a single wave [28]. It is a useful attempt to combine the simplistic maximum wave height and period measures in a more meaningful and complex characterisation. The output units are expressed as Joules per metre of wave front.

\[
E = 1961.H_w^2.T_w^2 \quad \text{Equation 4}
\]
It is important to note that the wave height ($H_w$) and wave period ($T_w$), variables are both squared. This means that a wave of low $H_w$, yet high $T_w$, could have the same energy value as a wave of low $T_w$ and high $H_w$. This fact can provide misleading results when comparing wave energies.

Kofoed-Hansen and Kirkegaard [51] describe a complex wash criterion which was originally developed for the operation of high-speed craft (HSC) in Danish waters, Equation 5. This criterion is a measure of wave effects upon a shore, and compares the wash from a HSC with the benchmark wash of a conventional vessel.

$$H_{hsc} = \beta^{3/2} \sqrt{T_c / T_{hsc}} \cdot H_c \quad \text{Equation 5}$$

$H_{hsc}$ is the maximum wave height (of the high speed craft), measured in a water depth of 3m, and $T_{hsc}$ its corresponding period. $H_c$ is maximum wave height (of the benchmark craft), measured in 3m deep water, and $T_c$ its corresponding period. $\beta$ being the decision parameter, usually taken as unity.

This criterion, while representing the concerns of shore goers and recreational vessels, can prove difficult to measure. This is mainly due to the 3m water depth requirement, but also due to the criticality of the wave as it enters shallow water. Fundamentally the local topography of the coast where the measurement occurs may significantly affect results.

A characterisation which fulfils most of the requirements for a suitable wash measure is the wave decay function, as prefaced in Chapter 3. It known that the wave decay coefficient ($n$), as stated in Equation 1, varies in shallow water, (as a function of $Fr_h$), from its deep water value of -1/3.

Another observed, yet less common wash characterisation is solitary waves or solitons, (as prefaced in Chapter 5). Of significance is that solitons are only generated by vessels travelling in shallow water, at or near the critical speed. Therefore it is not observed in the sub-critical (deep water), or super-critical speeds. This is significant in establishing solitons as a wash characterisation.

In considering the existing aforementioned measures, $H_w$, $T_w$, $Fr_h$, $a_{lead}$, while assisting in characterising wash, are too simplistic in nature and too restricted in scope. The more complex measures, ($E$, $H_{hsc}$, $\gamma$), while having greater scope, are either open to misinterpretation, too specific, or are not suitable for trans-critical analysis. An alternate view or perspective of the problem may provide a suitable measure to help further characterise vessel wash. This is now discussed.
Signal Analysis

By considering a wave in a mathematical context, (that is not as a physical entity but as a complex signal), a typical wave cut can be described as a transient signal. Such a signal can be analysed using modern signal analysis techniques.

Fourier analysis relies on the core assumption that the waveform being analysed is periodic. Wash signals are non-periodic or transient, and therefore have an important time domain component. The Fast Fourier Transform of a transient wave will produce a continuous spectrum in the frequency domain only and therefore will have limited application. Additionally choosing the length of wave elevation time signal to be analysed by Fourier analysis can be problematic, with the results being highly dependent on the sample length.

A mathematical tool for analysing transient signals uses the concept of wavelets. Wavelets allow a transient signal to be broken down into its constituent frequencies. Such frequencies, (and their relevant amplitudes), provide a unique identification of such a discrete signal, and its energy potential.

The term wavelet first appeared in Alfred Haar’s 1909 thesis [90], as “Haar Wavelets”. Following Haar there were some minor advances in wavelet analysis from the 1930s to the 1970s. However the next significant leap in understanding, (and application), was by Jean Morlet, and his later combined work with Alex Grossman [91]. Their concept was that a signal could be transformed into wavelet form and then transformed back into the original signal without loss of information. This is fundamental to the application of (wavelet based) data compression.

Following on, Mallat, [92], advanced wavelet analysis techniques through his work in digital signal processing. The current “state of the art” wavelet technique was originally developed by Daubechies [93]. Daubechies proposed a new class of wavelet, which has most significance within the field of digital filtering. By combining Daubechies and Mallat’s work, it is possible to transform signals, with minimal information loss using low computational requirements.

Fundamentally wavelet analysis is similar to Fourier analysis, with both methods breaking down time domain signals into their individual components and plotting them in the frequency domain. However where all the localised time information is lost in the FFT process, (i.e. the ability to describe when an event took place within the signal), wavelet analysis has the key benefit being of able to represent signals as a combined time-frequency representation.
Wavelet analysis output provides a time-scale frequency map of a signal, which enables the identification of time specific features. Clearly such a capability is ideal for analysing transient signals, and within this chapter wavelet analysis is utilised to attempt to identify key wash characteristics.

A detailed mathematical background of wavelets is beyond the scope of this chapter, but can be found within the following key references; [91 to 100].

**Physical Tests**

Two hull forms were tested, one a low wash type, the other a conventional fuller form. They tests were carried out in shallow water at the AMC MTB. The model test basin, the test programme, measurement and data acquisition, and hull forms utilised have been fully described in Chapter 2, and for the sake of brevity will not be repeated here.

**Results**

**Presentation**

Wavelet analysis provides both numerical and visual outputs. For this work the key numerical outputs utilised are peak spectral energy and its associated frequency. The visual outputs utilised are two dimensional (2D) and three dimensional (3D) combination plots.

Each 2D plot is a combination of two separate yet related sub-plots. The upper sub-plot is a standard longitudinal wave cut, of wave amplitude against time. The lower sub-plot is the associated wavelet analysis output of the longitudinal wave cut. Said output consists of: time (x-axis), plotted against frequency (y-axis), and is also plotted against spectral energy (z-axis).

For this work, and to facilitate comparison, the 2D wavelet analysis sub-plots have their spectral energy (z-axis) normalised to a unity value. This enables the form and peak energy values of the output to be reviewed easily.

The 3D plots represent the same information as the 2D plots but combined in three dimensional form. Additionally the 3D plots do not have the spectral energy (z-axis) normalised, but it is left in engineering units. This enables the relative magnitude as well as form of the spectral energy to be shown.

It should be noted for purposes of clarity that within the 2D plots the peak / maximum spectral energy is coloured red whilst the minimal spectral energy is coloured blue, (as per the legend bar). This colouring is carried over to the 3D plots; however the vertical scale of
each 3D plot is set to the maximum value for that series and not to each maximum on that plot.

Additionally it should be noted that all results presented are derived from longitudinal wave cuts, measured at wave probe 9, (Figure 6). With the exception of the results presented the section “Effect of time” which utilise probes 15, 16, 12, 17, 9, 18 and 9.

Review Method
The devised metrics for reviewing wavelet analysis results (plots + numerical) are;

(i) the value of the peak spectral energy.
(ii) the location of the peak spectral energy,
(iii) the frequency of the peak spectral energy,
(iv) the frequency range of the global spectral energy,
(v) the form of the global spectral energy.

Effect of Water Depth (h)
The effect of water depth was investigated utilising wavelet analysis for the NPL+ hull form. A matrix of four water depths, (h = 200 / 400 / 600 / 800 mm), and three Froude depth numbers, (Frₜ = 0.5 / 0.7 / 1.0), was tested. It should of course be noted that, for a fixed Frₜ value, over a range of water depths, the Frₕ value must vary. For the sake of brevity results from only one Froude depth number, (Frₜ = 0.7) have been reproduced.

From a review of 2D (normalised E) plots (Figure 37 to Figure 40) it is observed that, as the water depth progressively increases; (i) the average energy frequency reduces; (ii) the energy frequency range / band becomes narrower and (iii) the form of the energy becomes less intermittent or scattered. Additionally the location of the peak spectral energy (shown in red) stays relatively constant within the first wave packet.
Figure 37 – Effect of Water Depth - 2D Plot - $Fr_h = 0.7$ - $h = 200\text{mm}$

Figure 38 - Effect of Water Depth - 2D Plot - $Fr_h = 0.7$ - $h = 400\text{mm}$
6 Wavelet Analysis

Figure 39 - Effect of Water Depth - 2D Plot - Frh = 0.7 - h = 600mm

Figure 40 - Effect of Water Depth - 2D Plot - Frh = 0.7 - h = 800mm
From a review of the 3D plots (Figure 41 to Figure 44), a significant change in the peak spectral energy ($E$) is clear. This is in addition to the aforementioned changes in spectral energy average frequency and form.

**Figure 41 - Effect of Water Depth - 3D Plot - $Fr_h = 0.7$ - $h = 200$mm**

**Figure 42 – Effect of Water Depth - 3D Plot - $Fr_h = 0.7$ - $h = 400$mm**

78
Figure 43 – Effect of Water Depth - 3D Plot - Frh = 0.7 - h = 600mm

Figure 44 – Effect of Water Depth - 3D Plot - Frh = 0.7 - h = 800mm
Values of $E$ were measured for each case and the following was observed. For $Fr_h = 0.5$ and $Fr_h = 0.7$ (both sub-critical), as $h$ increases, so too does the $E$ value. However for $Fr_h = 1.0$ (critical) the $E$ value does not increase in the same manner.

As discussed in Chapter 5, when operating at or near the critical $Fr_h$, the measured wave height results are time dependant. Accordingly without a full time history of the event, any single result is essentially a snap-shot, which may or may not be at the maximum value. This was reported as “unsteadiness” due to soliton generation. This phenomenon would seem also to hold true when measuring $E$ at the critical $Fr_h$ number.

**Effect of Froude Depth Number ($Fr_h$)**

The effect of $Fr_h$ (for fixed water depth) was investigated utilising wavelet analysis for the NPL+ hull form. A matrix of three Froude depth numbers, ($Fr_h = 0.5 / 0.7 / 1.0$), and four water depths, ($h = 200 / 400 / 600 / 800$ mm), was tested. Again it should be noted that for a fixed $Fr_h$ value, over a range of water depths, the $Fr_L$ value must vary. For conciseness only one water depth ($h = 400$ mm) has been reproduced.

From a review of the 2D (normalised $E$) plots (Figure 45 to Figure 47) it is observed that as $Fr_h$ progressively increases: (i) the average energy frequency reduces and (ii) the energy frequency range / band is narrower. Similar trends were found for constant $Fr_h$ but with changing water depth, as per Chapter 4.
Figure 45 – Effect of Frh - 2D Plot - h = 400mm - Frh = 0.5

Figure 46 - Effect of Frh - 2D Plot - h = 400mm - Frh = 0.7
From a review of the 3D plots (Figure 48 to Figure 50), a significant change in $E$ value is clear. This is in addition to the aforementioned changes in spectral energy average frequency and form.

Values of $E$ were measured for each case and it was observed that, at all water depths, as the $Fr_h$ increased so too did the $E$ value. However it is noted that the $E$ values for $Fr_h = 1.0$ may be time-dependant (unsteady).
Figure 48 – Effect of Frh - 3D Plot - h = 400mm - Frh = 0.5

Figure 49 – Effect of Frh - 3D Plot - h = 400mm - Frh = 0.7
Wavelet analysis was used to investigate the effect of displacement for the NPL+ hull form. A matrix of two displacements, \( \frac{L}{\sqrt[3]{V}} = 8.5 / 9.2 \), three Froude depth numbers, \( \text{Fr}_h = 0.5 / 0.7 / 1.0 \), for one water depth \( h = 400 \text{ mm} \), was tested. For the sake of brevity only \( \text{Fr}_h = 0.5 \) is shown.

From a review of the 2D (normalised E) plots (Figure 51 and Figure 52), interestingly it is observed that the spectral energy characteristics, (i.e. average frequency, range and form), are very similar between displacements. This finding is common across all \( \text{Fr}_h \) tested. As shown in the 3D plots (Figure 53 and Figure 54) the form remains consistent with the change in displacement but the energy value increases with an increase in displacement.

A review of the measured peak spectral energy shows that, (for all \( \text{Fr}_h \) values), an increase in displacement leads to an increase in \( E \).

**Figure 50 – Effect of \( \text{Fr}_h \) - 3D Plot - \( h = 400 \text{mm} \) - \( \text{Fr}_h = 1.0 \)**
Figure 51 – Effect of $\Delta$ - 2D Plot – $h = 400$mm – Light $\Delta$ – $Fr_h = 0.5$

Figure 52 – Effect of $\Delta$ - 2D Plot – $h = 400$mm – Heavy $\Delta$ – $Fr_h = 0.5$
6 Wavelet Analysis

Figure 53 - Effect of \( \Delta \) - 3D Plot - \( h = 400 \text{mm} \) – Light \( \Delta \) – \( Fr_h = 0.5 \)

Figure 54 – Effect of \( \Delta \) - 3D Plot - \( h = 400 \text{mm} \) – Heavy \( \Delta \) – \( Fr_h = 0.5 \)
Effect of Hull Form
The effect of hull form was investigated for a fixed water depth \((h = 250\, \text{mm})\). Two hull forms (NPL+/ EH) were tested for five depth Froude numbers \((Fr_h = 0.5 / 0.7 / 1.0 / 1.3 / 1.5)\). From a review of the 2D and 3D plots it was observed that there was a small difference in spectral energy form between the two hulls. For brevity no results have been presented.

Effect of Time (Soliton)
The effect of time was investigated for the NPL+ hull form. The hull was run at a fixed water depth \((h = 200\, \text{mm})\), and for a fixed Froude depth number, \((Fr_h = 1.0)\).

In chapter 5, a comparison of the measured wave heights along the main longitudinal probe array (Figure 6) was utilised to determine (soliton) wave growth. It was believed that Wavelet analysis would capture the dynamics of the soliton cycle.

The 2D (normalised E) plots (Figure 55 to Figure 60) show that the average spectral energy frequency and its associated range / band are relatively similar. From close inspection it can be observed that there is a slight yet noticeable difference in the form of spectral energy across the probes. The measured peak spectral energy \((E)\) though shows little change in value, and there is no change in the associated peak frequency value.

Figure 55 – 2D Plot – Hull = NPL+, \(h = 200\, \text{mm}, Fr_h = 1.0\), Probe 13
Figure 56 – 2D Plot – Hull = NPL+, h = 200mm, Frh = 1.0, Probe 14

Figure 57 – 2D Plot – Hull = NPL+, h = 200mm, Frh = 1.0, Probe 11
Figure 58 – 2D Plot – Hull = NPL+, h = 200mm, Fr = 1.0, Probe 15

Figure 59 – 2D Plot – Hull = NPL+, h = 200mm, Fr = 1.0, Probe 16
Discussion

The devised methods for reviewing wavelet analysis results were successful in characterising vessel wash due to changes in; (i) water depth, (ii) Froude depth number, (iii) displacement and (iv) hull form.

With regard to the effect of displacement and the effect of hull form, it is noted that, in the deep water case [59], the EH and NPL+ hull forms have high $L/\Delta^{1/3}$ and $L/B$ ratios, which is a characteristic of their relatively low wave heights.

Additionally both hull forms are catamarans, and it is possible that a monohull would have significantly different wavelet characteristics. Accordingly (for these hull forms) wavelet analysis does not strongly identify the hull form variation. A systematic series of fuller-form mono hulls (e.g. the AMECRC Systematic Series [101]), may be more suitable for illustrating the influence of hull form on wash through wavelet analysis.

With regard to the effect of time (soliton), the wavelet analysis results are inconclusive. It is probable that Wavelet analysis of a complete soliton (time) event, (initiation – growth – shed), would show more clearly the change in spectral characteristics.
Concluding Remarks
The key benefit of wavelet analysis lies in its graphical output, which highlights features not clearly discernible from (traditional) analysis alone.

The 2D plots enable side by side comparison of the spectral form. The 3D plots clearly show the change in spectral magnitude as well as form.

These graphic outputs combined with the key wavelet analysis characteristic of being able to describe when events take place within a signal, make wavelet analysis a powerful and useful tool for wash characterisation.

Wavelet analysis has been shown to be a useful tool (metric) in characterising vessel wash. It has been able to capture the effects of water depth (fixed Fr_h), Fr_h (fixed h), displacement (fixed h), hull form, and time (soliton)

Further Work
An expected outcome of this wavelet analysis “proof of concept” work was the generation of more questions, and in turn the extension of the wash programme. Directly from this work there are four areas requiring further investigation.

Firstly wavelet analysis of a fuller-form monohull would further highlight the effects of displacement. It may also enable comparison between monohulls and catamarans. Secondly wavelet analysis on a complete systematic series (i.e. AMECRC), would enable a detailed review for changes in various hull form coefficients. Thirdly wavelet analysis of an entire soliton cycle may enable full characterisation and an alternate understanding of the event. Finally exploration of the effect of Fr_L in deep water, on vessel wash, utilising wavelet analysis.
7. When Is Water Shallow?

This research was originally published as;

“When is Water Shallow?”

Alex Robbins¹, Giles Thomas¹, Ian Dand², Gregor Macfarlane¹, Martin Renilson¹³,

¹ Australian Maritime College, Tasmania, Australia
²BMT Isis, Fareham, UK
³Higher Colleges of Technology, UAE,


For the avoidance of repetition the original paper has been modified for this thesis.
Paper Abstract

A master of a vessel must at all times know where their vessel is operating. Traditionally this is only thought of in the geographical sense; however there is a clear necessity, for safe vessel operations, that the master knows where their vessel is in the hydrodynamic sense. Water depth has profound effects on vessel performance and to know "When is Water Shallow?" is the key to successful vessel operation and wash mitigation. Accordingly, a series of characterisations is proposed to produce a more comprehensive definition of shallow water and hence provide greater operational understanding. Said characterisations cover the typical vessel performance indicators such as resistance, propulsion, manoeuvring etc., but also wash specific performance indicators such as wave angle, wave decay, soliton occurrence and spectral output.

Introduction

Currently there is no accepted unambiguous definition, (either numerical or descriptive), of when a vessel is affected by shallow water. The multiple physical variables involved make this a complex and subjective problem. Furthermore it is understood from existing works that shallow water wash (wash) is more complex than wash generated in deep water [14], and in turn it can have significantly different, (even fatal), outcomes for other water users [6].

With this in mind, the effects of shallow water, as presently known, are now briefly described with a view to characterising the various operating modes in shallow water more precisely.

Saunders, [42], in his key work on ship hydrodynamics, offers a definition of shallow (restricted) water; "A body of water is considered to be shallow when the boundaries are close enough to the ship to affect its resistance, speed, attitude, manoeuvring, and other performance characteristics as compared to its corresponding behaviour in a body of water of unlimited depth." Each of the performance criteria mentioned within Saunders definition can be traced to other independent works.

Saunders goes further to quantify shallow water, stating that a vessel can be considered to be in “restricted” waters if total vessel resistance (R_t) is increased by one percent, (1%).

It is understood that there are four operating zones, each with a different set of boundaries: (a) open water, (b) depth restricted, (c) width restricted, (d) combined depth and width restricted. The two most common shallow water restrictions, and those considered within this work, are (b) and (d). Typically when considering these restrictions the non-dimensionalised ratios of water depth to vessel draught, (h/T) and vessel cross-sectional area to channel cross-sectional area (blockage) ratio, (A_v/(b.W)), respectively are utilised.
When is Water Shallow?

From a fluid dynamics perspective, the flow around a vessel operating in shallow water can be described as a “restricted flow”. Following the continuity equation \([11]\), (i.e. constant mass flow rate), the fluid particles must accelerate past the restriction, (i.e. the vessel and the bottom). From Bernoulli’s principle \([102]\), this acceleration in turn leads to a local pressure reduction around the vessel, which is nominally described as the “venturi effect” \([103]\). Accordingly the increased local flow increases skin frictional (viscous) resistance. These viscous and pressure effects are increased when the flow is restricted by both depth and width, Figure 62.

The British Ship Research Association (BSRA) ship design manual \([83]\) lists the shallow water performance criteria and summarises them in a convenient plot, Figure 63. The manual describes three recognisable flow regimes and their effect on vessel performance: (a) **Infinite Depth**: essentially deep water with normal unrestricted flow occurring around the hull form; (b) **Intermediate Depth**: significant flow regime changes occurring which are noticeable, but do not have a significant effect on performance, (c) **Shallow Water**: the restriction of flow under the keel which has a dominant effect on vessel performance.
The key words within the BSRA definition here are “noticeable”, “significant” and “dominant”, which while somewhat subjective and non-specific however fit with it being a generic definition of shallow water. The BSRA plot (Figure 63) links various vessel performance criteria with the depth – draught ratio, (h/T). Additionally it indicates that vessel performance parameters are affected progressively as h/T decreases. A secondary finding from this figure is that not all performance parameters are affected at the same depth – draught ratio (h/T).

The BSRA figure is generic and descriptive in nature. It is probable that the performance parameters of long slender hull forms are affected differently to those of short squat hull forms, either by experiencing shallow water effects earlier (or later), or the vessel performance parameters being affected in a different order.

It should be noted that the h/T parameters in the physical tests presented in Chapters 3 to 6 have ranged from 8.0 to 1.8, which covers the deep, intermediate and shallow water ranges as presented by the BSRA manual.
Shallow Water Vessel Performance

Resistance

The most noticeable effect of vessel operations in shallow water is the change in resistance, which is most commonly felt as a loss in vessel speed [85]. Schlichtling's [69] experimental work provides an excellent semi-empirical guide as to the typical magnitude and form of said speed loss in open shallow water. This work was later extended by Lackenby [84] to incorporate more “realistic” areas of operation, and accounted for the correct skin frictional allowance, Figure 64.

![Figure 64 – Shallow Water Speed Loss Diagram](image)

Wave Pattern

Kelvin [14] first described the wave pattern of a moving point source at the surface. The Kelvin ship wave pattern(s) are well understood and are covered here only briefly. The deep water wave pattern consists of divergent and transverse wave systems. The cusp line of the divergent system intersects the sailing line at a fixed angle of 19°28'.

As the water depth is reduced, (for a given speed, for example), the divergent system angle progressively increases to approximately 90°, whilst the transverse system is ultimately lost with only the divergent system remaining. And as the water depth is reduced even further, (for a given speed), the wave angle progressively reduces with speed [14]. For clarity the simplified Kelvin wave pattern is shown in Chapter 4 in Figure 19.

96
Propulsion

It follows that as for vessel resistance, (due to the restricted flow around the hull), the performance of propellers will also be affected. Harvald [104] conducted open water and self-propulsion model experiments in shallow water for a bulk carrier form. These experiments showed that the propeller performance coefficients, (i.e. \( K_T \), \( K_Q \), \( \eta_0 \), and \( J \)), were affected and were non-linear functions of \( h/T \). For a given power, the typical reductions in propeller rpm are of the magnitude of 15% or more, Figure 65.

![Figure 65 – Propeller Open Water Efficiency][104]

Sinkage and Trim

Being a free floating body, a vessel will sink and trim according to the static and dynamic forces upon it. In shallow water the restricted flow causes a change in pressure around a vessel, causing, at low \( F_{Fr} \), a suction effect towards the boundary. This in turn causes the vessel to sink and trim by the head, the combination of which is known as squat [105]. The magnitude of ship squat is a direct function of vessel speed, \( h/T \) ratio and hull form. At high speed, a ship lifts and trims by the stern. In addition, the effect of initial trim can have a significant effect on the extent of which grounding at bow or stern may occur [106], Figure 66.
It should be noted that similar dynamic effects occur when vessels come close to a bank or even to other vessels (albeit in a lateral sense). This is known as interaction [107], and from a safety point of view can be just as significant as squat.

![Predicted Sinkage at F.P.](image)

**Figure 66 – Sinkage and Trim [108]**

**Manoeuvring**

In shallow water, it is appropriate to substitute the term “manoeuvrability” with “controllability”, as a vessel is considered to be controllable when the navigator is able to manoeuvre the vessel within acceptable limits [109]. Vessel controllability can be separated into three distinct areas of functions, being: (i) course keeping, (ii) manoeuvring and (iii) speed changing, all of which are affected by shallow water.

A vessel manoeuvring in shallow waters may be described as “sluggish” [109]. The added mass of the vessel increases, and the relationship between drift angle and side force is significantly altered, to the point where the vessel becomes uncontrollable. This “sluggishness” may also be explained partially by flow changes (propeller wake) onto the
vessel's rudder, leading to reduced hydrodynamic performance, as aforementioned above, (i, ii, iii).

An additional consideration is that as a vessel enters shallow water its resistance increases and the vessel slows (for a constant power or shaft speed). As a vessel's manoeuvring characteristics are directly a function of speed, this speed reduction has an added effect on vessel controllability. Dand [110] showed that ship handling characteristics (manoeuvring), as well as vessel resistance are affected by restrictions in water depth and width. Typically, (for a given speed), a vessel's turning circle increases as water depth decreases, Figure 67.

![Figure 67 - Turning Circle Manoeuvre](image)

**Motions**
As vessel resistance, trim, propulsion and manoeuvring are affected by shallow water, so too are vessel motions. As mentioned the reduced under keel clearance creates a suction type effect on the hull form which increases “hydrodynamic damping” [111]. This in turn leads to a reduction in vessel motions, (i.e. pitch, heave and roll), [112]. The magnitudes of these reductions are a direct function of vessel heading, speed, and water depth, (see Figure 68).
A summary of shallow water on vessel performance is shown below in Table 6.

<table>
<thead>
<tr>
<th>Characterisation</th>
<th>Hydrodynamic Effect</th>
<th>Outcome on Vessel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance (friction)</td>
<td>Skin friction increase</td>
<td>Speed decrease</td>
</tr>
<tr>
<td>Wave pattern</td>
<td>Local pressure change</td>
<td>Wave angle change</td>
</tr>
<tr>
<td>Propulsion</td>
<td>Modified wake</td>
<td>Loading change</td>
</tr>
<tr>
<td>Squat</td>
<td>Hydrodynamic suction</td>
<td>Draught and trim increase</td>
</tr>
<tr>
<td>Manoeuvring</td>
<td>Boundary back flow</td>
<td>Reduced responsiveness</td>
</tr>
<tr>
<td>Motions</td>
<td>Hydrodynamic added mass</td>
<td>Dampened motions</td>
</tr>
</tbody>
</table>

Table 6 – Shallow Water Vessel Performance
Wash Performance

The previous section has shown that the effect of shallow water on vessel performance is well known. However the effect of shallow water on wash-specific performance parameters is less well understood. In this section, we consider these performance parameters and extend previous work in deep water to water of limited depth with the intention of providing a more precise determination of various types of shallow water.

Within Chapter 2 deep water wash performance parameters, or characterisations, were assessed, i.e. wave height, wave period, wave energy etc. This assessment evaluated their suitability for use in deep and shallow water. From this review additional shallow water wash characterisations were recommended which fully describe wash across the trans-critical range. These new characterisations better define trans-critical wash and in turn should enable effective wash mitigation strategies to be developed.

Operational Conditions

With respect to vessel performance two operational conditions, (deep and shallow water), have been considered. However, with respect to vessel wash, the shallow water condition can be described more precisely. This is significant as knowing which shallow water condition a vessel is operating in will assist in developing effective mitigation strategies.

Following on from Havelock's work [70], the onset of shallow water effects is typically defined in terms of Froude depth number (Fr_h). It has been generally considered that the deep water condition covers the Fr_h range up to 0.5, after which the vessel can be considered to be operating in shallow water. From Havelock's work, three discreet shallow water operational conditions can be defined; (i) sub-critical, (ii) critical, and (iii) super-critical.

The deep and shallow water operational condition ranges are typically specified in terms of Fr_h, providing “ball park” estimations. However such a definition is a simplification of a highly complex problem, since each operational condition cannot be so clearly defined using a single parameter.

It should be understood that there are no set numerical boundaries between conditions, but rather transitional zones in between identifiable states. Accordingly it is recommended that outcome- or evidence-based classification, (which varies between vessel type and environment), rather than that based on simple (empirical) numerical formulae.
Wash Characterisations

The following key wash characterisations have been investigated and are proposed as suitable for defining trans-critical wash and boundaries of shallow water zones.

Leading Wave Angle

The change of vessel wave pattern, (i.e. transverse and divergent wave systems), as a function of $F_{rh}$, has been covered earlier within this Chapter. In addition to this characteristic is the angle of the leading wave ($\alpha_{lead}$) within the divergent wave system. As discussed in Chapter 4 $\alpha_{lead}$ is a key characterisation for trans-critical wash, being simple to measure and understand. For completeness a simplified version of Figure 20 (Chapter 4) is included here as Figure 69.

![Figure 69 - Leading Wave Angle ($\alpha_{lead}$) as a Function of $F_{rh}$](image)

Wave Decay

The decay of the leading wave has been covered in Chapter 3. To summarise; (i) a vessel’s maximum wave height ($H_w$), can be calculated for a given offset from sailing line ($y$), once gamma ($\gamma$), in Equation 1 is known; (ii) the deep water decay coefficient ($n$) is constant, ($n=-1/3$); (iii) for shallow water it is known that ($n$) varies with $F_{rh}$.

Therefore the decay coefficient ($n$) can be utilised as a characterisation for trans-critical wash. As an example, the decay rate for various FrL as a function of $h / L$ is shown in Figure 70.
When is Water Shallow?

Figure 70 – Leading Wave Decay (n) as a Function of Depth / Length Ratio

Soliton

As previously discussed, a soliton is a single non-dispersive wave with no preceding (or following) trough. Solitons are cyclical and time dependant in nature. John Scott Russell first observed the “wave of translation” (i.e. soliton) in 1844 [18]. It should be clarified that, for steady conditions, multiple individual solitons will be continuously formed ahead of the vessel, Figure 71 and Figure 72.

Within Ertekin’s [19] experiments, solitons were observed occurring over a range of Froude depth numbers from 0.9 to 1.2, (not just at the critical value). It was further reported that, for both measured and numerical experiments, the resistance oscillated about a mean value, with a period equal to that of soliton generation. This is confirmation that solitons are time-dependant in nature.

In Chapter 5 it was determined that for a vessel travelling near the critical depth Froude number, a time dependant “unsteadiness” was present within the results, as a precursor to a soliton forming.
Figure 71 – Solitons Generated from a Monohull in the AMC Towing Tank

This photo shows several solitons generated by a model of a full form vessel operating in shallow water. The solitons propagate ahead of the model, but are 'disturbed' close to the sidewalls of the tank as there is a 45 degree fillet in the corners of the tank (approximately equal to the water depth of 100mm). As can be seen, this has caused the solitons to form their own diverging wave system at the tank boundary, (i.e. non-dispersive waves creating a dispersive wave system).
Wavelet Analysis

The technique of spectral wavelet analysis of vessel wash has been covered in Chapter 6. Wavelet analysis is similar to Fourier analysis, with both methods breaking down time domain signals into their individual components and plotting them in the frequency domain. However, whereas in the Fast Fourier Transform process all the localised time information is lost, wavelet analysis has the key benefit of being able to describe when an event took place within the signal.

Wavelet analysis provides both numerical and visual outputs. The visual outputs, are two dimensional (2D) and three dimensional (3D) combination plots. Both plot types represent signals as a combined time-frequency.

Each 2D plot (Figure 73) is a combination of two separate yet related sub-plots. The upper sub-plot is a standard longitudinal wave cut, of wave amplitude as a function of time. The lower sub-plot is the associated wavelet analysis output of the longitudinal wave cut. The 3D plot (Figure 74) contains the same information as the 2D plot however it is presented in a three dimensional form.

Wavelet Analysis Metrics

The suggested metrics for reviewing wavelet analysis results are;

i. the value of the peak spectral energy.
ii. the location of the peak spectral energy,
iii. the frequency of the peak spectral energy,
iv. the frequency range of the global spectral energy,
v. the form of the global spectral energy

Figure 72 – Longitudinal Wave Cuts Showing Soliton Development [22]
7 When is Water Shallow?

Figure 73 – Wavelet Analysis – Typical 2D plot [113]

Figure 74 – Wavelet Analysis – Typical 3D Plot [113]
Characterisation Summaries

Considering the aforementioned wash characterisations, which are good engineering approximations, the following operational zone summaries can be made for all water depths.

Deep Water

The deep water operational zone can be characterised as; (in no order of significance),

- Froude depth number < 0.5
- Transverse and divergent wave systems both present;
- Leading wave angle of 19° 28’ (cusp locus line intersection);
- Divergent wave system decays at $n = -\frac{1}{3}$;
- Wave system is dispersive;
- No solitons are present;
- Wavelet analysis metrics (i) to (v), above, are constant with time;
- Vessel performance¹, (resistance, trim, heave, and squat etc.), constant with time;

By definition, if any one of these conditions is not met, then the vessel can be considered to be operating in shallow water.

Shallow Water Sub-Critical

The shallow water sub-critical operational zone can be characterised as:

- Froude depth number < 0.5 but > 1.0
- Transverse and divergent wave systems present, with the transverse system diminishing as $Fr_h$ increases;
- Leading wave angle increasing from 19° 28’ towards a maximum value (approx. 90°);
- Leading wave decay rate variable, (i.e. not constant at $n = -\frac{1}{3}$);
- The wave system transforming from a dispersive system to a combined dispersive/non-dispersive system;
- No solitons are present;
- Wavelet analysis metrics initially constant at low $Fr_h$ but becoming time dependant (variable) as $Fr_h$ increases;
- Vessel performance¹ differs from that in deep water;

¹ Vessel performance is not a wash characterisation but is a valuable co-dependent indicator
Shallow Water Critical
The shallow water critical operational zone can be characterised as:

i. Froude depth number $\equiv 1.0$
ii. Critical wave system present;
iii. Leading wave angle at a maximum value (approx. 90°);
iv. Leading wave decay rate is variable with $Fr_h$, (i.e. not constant at $n = -\frac{1}{3}$);
v. Wave system is non-dispersive;
vi. Solitons are generated;
vii. Wavelet analysis metrics vary with time, indicating non-dispersive conditions;
viii. Vessel performance differs from that in deep water;

Shallow Water Super-Critical
The shallow water Super-critical operational zone can be characterised as:

i. Froude depth number $> 1.0$
ii. Divergent wave system only present;
iii. Leading wave angle decreasing from the maximum value;
iv. Leading wave decay rate variable with $Fr_h$, (i.e. not constant at $n = -\frac{1}{3}$);
v. The wave system transforming from a non-dispersive to a dispersive system;
vi. No solitons present;
vii. Wavelet analysis metrics becoming less time dependant (variable) as $Fr_h$
     increases (i.e. transforming from non-dispersive to a dispersive system);
viii. Vessel performance differs from that in deep water;

The above deep water and shallow water characterisations have been summarised in Table 7.
### Operational Zone

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Froude Depth Number</td>
<td>Fr_h &lt; 0.5</td>
<td>0.5 &lt; Fr_h &lt; 1.0</td>
<td>Fr_h ≥ 1.0</td>
<td>Fr_h &gt; 1.0</td>
</tr>
<tr>
<td>Divergent Wave System</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Transverse Wave System</td>
<td>Yes</td>
<td>Diminishing</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Leading Wave Angle</td>
<td>Constant @19° 28'</td>
<td>19° 28' ≤ θ ≤ 90°</td>
<td>θ ≥ 90°</td>
<td>90° ≥ θ</td>
</tr>
<tr>
<td>Leading Wave Decay</td>
<td>Constant @ -1/3</td>
<td>Variable - f(Fr_h)</td>
<td>Variable - f(Fr_h)</td>
<td>Variable - f(Fr_h)</td>
</tr>
<tr>
<td>Wave System Dispersive</td>
<td>Yes</td>
<td>Diminishingly</td>
<td>No</td>
<td>Increasingly</td>
</tr>
<tr>
<td>Solitons</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Spectral (WA)</td>
<td>Constant</td>
<td>Variable with time</td>
<td>Variable for fixed Fr_L + Variable with time</td>
<td>Variable with time</td>
</tr>
<tr>
<td>Performance</td>
<td>Steady</td>
<td>Increasing</td>
<td>Oscillating</td>
<td>Reducing</td>
</tr>
</tbody>
</table>

Table 7 – Wash Characterisation Summary Table
Concluding Comments

For safe vessel operations a master of a vessel must at all times know where his vessel is operating, in both the geographical and hydrodynamic sense.

By deduction if one of the aforementioned deep water characteristics is not present, the vessel can be assumed to be operating in shallow water. The question “*When is water shallow?*” has been answered.

Furthermore from the characterisations it is possible to further classify shallow water, into sub-critical, critical, and super-critical operational zones

These characterisations while providing provide better definition of shallow water also have fundamental implications for safe vessel operations.
8. **Summary, Conclusions and Further Work**

**Summary**

Vessel generated wash, within the littoral zone, has been shown to be a clear and present hazard, posing significant risk to public safety and the environment.

Due to the lack of available information on shallow water wash it was determined that physical testing was required to clarify this complex phenomenon. Furthermore that the outcomes from this testing should be simplified characterisations, which could be utilised by naval architects, operators and regulators to mitigate the problem of wash pollution.

As prefaced within its title; "Shallow Water Catamaran Wash – Simple Characterisations for a Complex Phenomenon", this thesis explored vessel-generated wash in shallow water, with the specific goal of simplifying this highly complex phenomenon into user friendly characterisations.

To achieve this, a “thesis by publication” approach was adopted comprising five published journal papers, which formed the basis of Chapters 3 to 7. The first three papers, (Chapters 3, 4 and 5), exploit what are essentially existing deep water wash characterisations extended into shallow water. The fourth paper, on wavelet analysis, is both a new and novel approach to the existing problem of characterising wash. Fifth and final paper (Chapter 7), “When is Water Shallow?”, summarises the findings of all earlier papers, and contextualises them alongside established shallow water knowledge. This is not only significant in meeting the projects goal but also in establishing the relevance of this thesis.

Within Paper 1 (Chapter 3), two catamaran hull forms were tested in shallow water across the trans-critical range to investigate the effect of water depth on the decay of vessel wash. The re-application of the deep water decay method into shallow water established that the decay coefficient is a function of water depth (Fr_h). This outcome proves that that decay can be utilised as a simple method for the prediction of catamaran wash, and also is a valid characterisation. Furthermore the results confirmed that decay coefficient is independent of (catamaran) hull form and displacement.

Another important finding was that across the trans-critical range, there was significant superposition of competing wave systems. As a result the primary deep water measure of
maximum wave height provides erroneous and unreliable results. Therefore its use as a shallow water wash characterisation was limited. However it was also determined that a more reliable and easily identifiable shallow water metric was the bow or leading wave.

The second round of physical tests, outlined in the second paper (Chapter 4), built upon the work done in the first paper. The hull forms were again tested in shallow water across the trans-critical range, with the focus on the leading wave angle. For these tests the probe array was modified to capture the leading wave angle with greater resolution.

The form of the curve of the measured bow wave angle as a function of \( \text{Fr}_h \), closely corresponds to that predicted by theory. However, significantly, the peak value was found to occur closer to \( \text{Fr}_h =0.9 \) than 1.0. This was assumed to be due to non-linear viscosity effects, not accounted for in the theory. Importantly the form of the curve is independent of hull form or depth – draught ratio, and so can be utilised as a shallow water wash characterisation.

A review of results from the first and second physical test programmes highlighted some unexpected findings, specifically related to wave height and decay in the near-critical region. Accordingly the third test programme investigated the effect of starting accelerations and soliton formation as presented in Chapter 5. These tests determined that, as the critical number is approached, it is most likely that the formation of solitons, not starting accelerations, are the cause of the observed unsteadiness. Unsteadiness and solitons were shown to be a feature of near critical vessel behaviour, and therefore a wash characterisation.

Furthermore, due to the time-dependant nature of near-critical unsteadiness, a time stamp should be included with the results as a reference to indicate when, in the unsteadiness cycle, the wash measurement was taken. Significantly all previous wash measurements, both model and full scale, taken around the critical \( \text{Fr}_h \), will have been time-dependant (\( i.e. \) unsteady).

The shallow water wash characterisations outlined in the first three papers, in addition to existing wash characterisations (Froude depth number, divergent and transverse wave systems), were believed to be simplistic and not fully descriptive of this dynamic region. A characterisation was required which could clearly define the features of shallow water in a simple way. It was recognised that a longitudinal wave cut can be considered as a transient signal and in turn it would be open to signal analysis techniques, specifically wavelet analysis.

The fourth paper (Chapter 6) provides proof-of-concept that wavelet analysis is successful in characterising vessel wash. The wavelet methodology allows differentiation of various hull forms from their wave patterns alone. The key benefit of wavelet analysis lies in its graphical
output, which highlights features not clearly discernible from (traditional) numerical values alone. This technique is a “new and novel” approach to an existing problem and also meets both the project’s goals.

In keeping with the second goal of this thesis, the findings from the first four papers were summarised in the fifth, for deep and shallow water, (sub-critical, critical, super-critical), conditions. Additionally these characterisations were placed within the global context of existing vessel performance indicators, (resistance, propulsion and manoeuvring).

By deduction, if one of the aforementioned deep water characteristics is not present, the vessel can be assumed to be operating in shallow water. Furthermore from the characterisations it is possible to further classify shallow water, into sub-critical, critical, and super-critical operational zones. This final paper (Chapter 7) answers the question “When is water shallow?” Furthermore these characterisations, while providing a better definition of shallow water, also have fundamental implications for safe vessel operations.

As demonstrated, the thesis achieves its goal of simplifying the highly complex phenomenon of shallow water wash into user-friendly characterisations. The wash characterisations shown in Table 7 provide naval architects, operators and regulators alike a convenient summary of said characterisations. Such information increases the understanding of operators and regulators of vessel wash in shallow water and in turn effects hazard reduction.

Conclusions

As a result of the study described in this thesis, the following main conclusions are drawn:

- Shallow water hydrodynamic effects are complex, especially in the trans-critical regime.

- None of the existing metrics used to define shallow water are able to characterise such effects adequately.

- Nine characterisations are proposed for the correct assessment of trans-critical vessel wash. The use of these characterisations will lead to better understanding (of trans-critical vessel wash) and in turn effect hazard reduction.

- Wavelet analysis has been shown to be a useful, and potentially powerful, tool for the study of shallow water effects.
Further Work

The experimental programmes were restricted by the physical parameters of the model test basin, (a typical experimenter’s problem). Ideally the basin should have been longer, specifically so as to enable a complete soliton cycle to develop. Also it would have been desirable to investigate the effects of blockage within the test programmes. To enable this (false) movable basin walls would be required.

The “lessons learnt” from the test programmes are evident in the final wave probe array, the synchronised photography, and the model’s bow position being recorded on each wave trace. Additionally there was much development work done on data handling and data analysis techniques. This included semi-automated routines for the post processing of raw data and complex interlinked spread sheets enabling speedy effective data analysis. These lessons learnt resulted in reduction in analysis time, an increase in data quality, and overall confidence in the results.

An expected outcome of this thesis was the generation of more questions, and the requirement for future work. Immediately there are three areas requiring further investigation.

The testing and wash measurement of;

(i) a full formed monohull, of equivalent displacement to the NPL+ hull. This would enable direct comparison between monohulls and catamarans.

(ii) an existing systematic series (i.e. AMECRC). This would enable a detailed review of the changes in wash due to hull form variation.

(iii) an entire soliton cycle, at varying water depths. This would shed light on this highly dynamic, yet essentially unknown area.

Each of the outlined wash characterisations would provide further definition in the above areas; however it is wavelet analysis which potentially provides the greatest opportunity for further advancement of knowledge. It is recognised that further development of the wavelet analysis technique is required to fully exploit its potential. An area of specific application for wavelet analysis is vessel identification which has use in multiple fields, not least defence.

Furthermore CFD may develop sufficiently to enable this works test programmes to be completed numerically. If validated, this would be a significant time and resource saving over conventional physical tests.
9. References


2. Infrastructure Australia, “Shipping Australia’s submission to the Infrastructure Australia to assist with the audit of existing infrastructure and development of priorities for investment in infrastructure in the future.” Public Submission 2008 / 2009, #184.


9 References


9 References


10. **Appendix CD**

The enclosed data CD contains additional information relevant to the thesis.

A. Physical Test Programmes

B. Numerical Analysis

C. Photos

D. Published Papers