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A Re-examination of the Real Interest Parity Condition Using Threshold Cointegration

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Abstract: Threshold cointegration is employed in this study to test the real interest parity condition between the UK and the US. Evidence supports the asymmetric adjustment of real interest rates. The threshold error correction models indicate that negative deviations from long run real interest parity are eliminated faster than positive deviations.

JEL Classification: E 43, F36, F41

Keywords: real interest parity, threshold cointegration, threshold error correction, asymmetric adjustment, non-linear adjustment.

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1 Introduction

This paper employs threshold cointegration to investigate the real interest parity condition between the UK and the US. Pippenger and Goering (1993), Balke and Fomby (1997), Enders and Granger (1998), Enders and Siklos (2001), Hansen and Seo (2002) show that conventional unit root and cointegration tests exhibit low power in the presence of non-linear adjustment towards long run equilibrium. Hence, the main purpose of this paper is to see if the adjustment of the real interest rate towards long run equilibrium is asymmetric. The asymmetric adjustment of real interest rates suggests that a cointegrating relationship exists between real rates during certain periods and not during others. The rest of this paper is structured as follows. Section 2 includes a discussion of the approach applied, Section 3 examines the properties of the data set, the results of the analysis are presented in Section 4 and conclusions drawn in Section 5 with policy implications.

2 Real Interest Parity and Threshold Cointegration

A test of real interest parity constitutes estimating the following equation:

$$r_t = \alpha + \beta r_t^* + \varepsilon_t \tag{1}$$

where r_t is the real interest rate in the reference country; r_t^* is the real interest rate in the foreign country and ε_t is the stochastic error term. The existence of real interest parity implies that the ε_t series is a stationary process. Enders and Granger (1998) and Enders and Siklos (2001) put forward a test for a non stationary series against an alternative of asymmetric adjustment where the process is a two regime Threshold Autoregressive (TAR) or Momentum TAR (M-TAR) model. This paper employs only the TAR model as it was found to be a better specification for the data used. Therefore, following the approach of Enders and Granger and Enders and Siklos, the regression residuals from equation (1) are estimated in the following manner:

$$\Delta\hat{\varepsilon}_t = I_t \rho_1 \hat{\varepsilon}_{t-1} + (1 - I_t)\rho_2 \hat{\varepsilon}_{t-1} + \nu_t \tag{2}$$

where

$$I_t = \begin{cases} 1 & if \quad \hat{\varepsilon}_{t-1} \geq \tau \\ 0 & if \quad \hat{\varepsilon}_{t-1} < \tau \end{cases} \tag{3}$$

The value of the threshold is denoted by τ. What this implies is that if $\hat{\varepsilon}_{t-1} \geq \tau$, I_t takes on a value of one and the speed of adjustment in equation (2) is ρ_1. If on the other hand $\hat{\varepsilon}_{t-1} < \tau$, I_t takes on a value of zero and the speed of adjustment is ρ_2. If $|\rho_1| > |\rho_2|$, the adjustment process is faster for $\hat{\varepsilon}_{t-1} \geq \tau$ than $\hat{\varepsilon}_{t-1} < \tau$. Enders and Granger have computed critical values for the null of a unit root, that is, $\rho_1 = \rho_2 = 0$, against the TAR alternatives. The F statistic for the null hypothesis that $\rho_1 = \rho_2 = 0$ using the TAR model is denoted by Φ_u. A sufficient condition for the $\{\hat{\varepsilon}_t\}$ series to be stationary is -2 < ($\rho_{1,} \rho_2$) < 0. If $\rho_1 = \rho_2$, then equation (2) is equivalent to the Dickey Fuller test. The TAR models are estimated in Section 4 using an estimated value for τ. A τ value is estimated using Chan's (1993) method. This procedure is explained in Section 4.

3 Data

The data used are three month Euro Dollar Deposit Rates for the US and the UK. All data are obtained from *Global Financial Data.* This ensures that the assets are comparable in terms of risk and tax treatment (see Siklos and Granger 1997). The data covers the period 1980.7 to 2005.2. Real interest rates are calculated as the nominal rate of interest less the rate of inflation, i-π.

Table 1 presents the Augmented Dickey Fuller (ADF -1979), Phillips-Perron (PP – 1988), and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS - 1992) test statistics for unit roots. The results suggest that both interest rate series are non stationary in levels and stationary in the first differences.

Table 1: Unit Root Tests

	ADF	PP	KPSS
r_{US}	-2.14	-1.47	1.53***
r_{UK}	-2.01	-2.24	1.44***
$\Delta\ r_{US}$	-15.40***	-15.37***	0.04
$\Delta\ r_{UK}$	-4.32***	-21.21***	0.06

Critical values at 1%, 5% and 10% levels ADF and PP: -3.45, -2.87, -2.57
KPSS 1%, 5% and 10% levels: 0.739, 0.463, 0.347 (H_o = stationarity)

4 Empirical Results

Threshold Cointegration:

Chan's (1993) procedure is used to calculate an estimate for the threshold. According to Chan, in order to obtain a consistent estimate of τ , the estimate of τ must lie between the maximum and minimum values of the series. The estimate of τ is computed as follows. The series is ranked. Next, the highest 15% and lowest 15% of the series, is removed. Of the remaining 70% of the data points, each one has the potential to be the threshold. The estimates for the threshold parameters for each model are selected so that the sum of squared residuals is minimized for each equation. Having followed this procedure, the selected τ value for r_{UK-US} is 0.42849. Table 2 reports cointegration test results for the equation with a consistent estimator of the threshold.

Table 2: Threshold Cointegration with Estimate of Threshold

$r_{UK} - r_{US}$

$$\Delta\hat{\varepsilon}_t = \underset{(2.885)}{0.0078}\ I_t\hat{\varepsilon}_{t-1} + \underset{(3.698)}{0.0122}\ (1 - I_t)\hat{\varepsilon}_{t-1} + \nu_t \quad (4)$$

Φ_U = 59.25*** $\rho_1 = \rho_2$: 0.00 $\tau = 0.42849$

AIC: 251 SBC: 255

Notes: t statistics reported in parenthesis
critical values for threshold unit roots: 10%, 5% and 1% levels respectively: -5.11, -6.03, 8.04
$\rho_1 = \rho_2$ denote symmetric adjustment and the values expressed are the p values of symmetric adjustment.

Observe that symmetric adjustment, that is $\rho_1 = \rho_2$, is rejected at the 1% level in the above Table. Real interest parity therefore appears to hold given the non-linear adjustment in interest rates. The estimates for ρ_1 and ρ_2 are 0.0078 and 0.0122 respectively, suggesting that negative deviations from equilibrium adjust faster to long run equilibrium, at a rate of 1.2%, compared to positive deviations from real interest parity which adjust at a rate of 0.7%.

Threshold Error Correction

If real interest parity holds in an asymmetric model, an error correction model can be used to check the short run dynamics of the time series. The general asymmetric error correction model for the real interest parity condition given by equation (1) can be represented as:

$$\Delta r_t = \vartheta + \delta_{11}\ ec^{+}_{t-1} + \delta_{12}\ ec^{-}_{t-1} + \alpha_{11}(L)\Delta r^{*}_{t-1} + \alpha_{12}(L)\Delta r_{t-1}$$

where ϑ is a constant and ec^{+}_{t-1} and ec^{-}_{t-1} are the error correction terms. The estimated coefficients on ec^{+}_{t-1} and ec^{-}_{t-1} determine the rate at which positive and negative deviations from real interest parity adjust to long run equilibrium.

Using the consistent estimate of the threshold, ec^{+}_{t-1} and ec^{-}_{t-1} are estimated based on the cointegrating relationship between the r_{UK} and $r_{US.}$ OLS is used to estimate the long run relation. This yielded: $r_t = -3.76 + 0.72 r*$. Using these estimates ec^{+}_{t-1} and ec^{-}_{t-1} have been calculated as follows:

$$ec^{+}_{t-1} = I(r_{t-1} - 0.72\ r^{*}_{t-1} + 3.76);\ ec^{-}_{t-1} = (1-I)(r_{t-1} - 0.72 r^{*}_{t-1} + 3.76);\ \alpha_{ij}(L)$$

is a 4th order polynomial in the lag operator L. The lag length is selected according to the AIC criteria. Equations (4) and (5) are based upon these estimates. The estimated coefficients for all variables are reported in Table 3. For purposes of evaluating the error correction terms, equations (4) – (5) report the coefficients on the error correction terms only.

Reported below are the estimated error correction models with t statistics reported in parenthesis.

$$\Delta r_{t_{UK}} = \vartheta_3 - \underset{(-0.23)}{0.0079}\ ec^{+}_{t-1} - \underset{(2.86)}{0.1559}\ ec^{-}_{t-1} + \alpha_{11}(L)\Delta r^{*}_{t-1\ US} + \alpha_{12}(L)\Delta r_{t-1\ UK} \tag{4}$$

$$\Delta r_{US} = \vartheta_4 - \underset{(-0.25)}{0.0061}\ ec^{+}_{t-1} - \underset{(2.72)}{0.1055}\ ec^{-}_{t-1} + \alpha_{11}(L)\Delta r^{*}_{t-1\ UK} + \alpha_{12}(L)\Delta r_{t-1\ US} \tag{5}$$

Equations (4) and (5) which are based upon the regression of $r_{UK} - r_{US}$, indicate that negative deviations from real interest parity are eliminated faster than positive deviations. The point estimates for equation (4) suggest that if there is a unit positive deviation from interest parity, it is corrected at a rate of 0.79% in one month while a unit point negative deviation from interest parity is corrected at a rate of 15% in a month.

The estimates in equation (5) indicate that 0.61% of the discrepancy of a positive deviation from real interest parity is eliminated in one period while a negative deviation from interest parity is corrected at a faster rate of 10.55%. The negative deviations are significant in both equations.

Table 3: Error Correction Models

$$\Delta r_{t_{UK}} = \vartheta_3 - 0.0079\ ec^{+}_{t-1} - 0.1559\ ec^{-}_{t-1} + \alpha_{11}(L)\Delta r^{*}_{t-1\ US} + \alpha_{12}(L)\Delta r_{t-1\ UK}$$

Variable	Coefficient	t-Statistic
ec^{+}_{t-1}	-0.007931	-0.231895
ec^{-}_{t-1}	-0.155894	2.864021
$\Delta r_{UK(t-1)}$	-0.146601	-2.510685
$\Delta r_{UK(t-2)}$	0.086769	1.472798
$\Delta r_{UK(t-3)}$	-0.177547	-3.036343
$\Delta r_{UK(t-4)}$	-0.076785	-1.330569
$\Delta r_{US(t-1)}$	0.008046	0.101945
$\Delta r_{US(t-2)}$	0.070443	0.898343
$\Delta r_{US(t-3)}$	0.122532	1.576489
$\Delta r_{US(t-4)}$	-0.049507	-0.659832
ϑ_3	0.123482	1.487970

$\chi^2_{sc} = 4.52$ $\chi^2_{n} = 2.29$ $\chi^2_{hs} = 0.23$

$$\Delta r_{US} = \vartheta_4 - 0.0061\ ec^{+}_{t-1} - 0.1055\ ec^{-}_{t-1} + \alpha_{11}(L)\Delta r^{*}_{t-1\ UK} + \alpha_{12}(L)\Delta r_{t-1\ US}$$

Variable	Coefficient	t-Statistic
ec^{+}_{t-1}	-0.006113	-0.247583
ec^{-}_{t-1}	-0.105487	-2.724134
$\Delta r_{US(t-1)}$	0.003632	0.063338
$\Delta r_{US(t-2)}$	-0.088773	-1.578194
$\Delta r_{US(t-3)}$	-0.013870	-0.248084
$\Delta r_{US(t-4)}$	-0.056810	-1.051342
$\Delta r_{UK(t-1)}$	0.084245	1.986384
$\Delta r_{UK(t-2)}$	-0.014009	-0.327430
$\Delta r_{UK(t-3)}$	-0.038802	-0.905432
$\Delta r_{UK(t-4)}$	0.023864	0.566266
ϑ_4	-0.139194	-2.373129

$\chi^2_{sc} = 6.52$ $\chi^2_{n} = 1.30$ $\chi^2_{hs} = 0.46$

5 Policy Implications and Conclusions

The results suggest that real interest parity holds between the Euro rates of the UK and the US when asymmetric adjustment is taken into account. Siklos and Granger (1997) show that an equilibrium relationship can change if one country that has adopted an inflation targeting regime has close ties with another that does not follow an inflation targeting policy. The UK introduced a policy of inflation targeting in 1992. The US has not yet adopted a policy of inflation targeting. This perhaps is the reason for the asymmetric adjustment to long run real interest parity.

The estimates of the cointegrating error correction models indicate that negative deviations from interest parity are eliminated faster than positive deviations. In recent times the UK and the US have both experienced low real rates, however, this has not led to stronger growth. How can this be explained in the context of these results? One explanation is that negative shocks in the UK have led to a widening of the negative output gap offsetting the stimulating effects of low real interest rates. Another possible explanation is that the asymmetric adjustment in interest rates has led to asymmetric information in credit and financial markets and as pointed out by Rajan (2005), in the presence of low real rates of interest, investors can under price risk leading them to undertake increased speculative investment. Under such circumstances the Bank of England is more likely to intervene in order to correct a negative shock to restore the economy back to long run equilibrium. In conclusion, the results suggest that real interest parity holds between the US and the UK during some periods and not in others. This implies that the two countries can pursue independent monetary policies during certain periods and not during others.

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