In: DF Chapman, DA Clark, KL Macmillan, DP Nation (Editors). Dairy Science 2007, Meeting the Challenges for Pasture-Based Dairying', *Proceedings of the 3rd Australasian Dairy Science Symposium*, The University of Melbourne, Victoria, Australia, 3:423-430 (2007)

Predictive characteristics of lactation models for pasture-based Holstein Friesian dairy cows

S.A. ADEDIRAN[1], J.R. ROCHE[2], D.J. DONAGHY[2], R. RAWNSLEY[2], M. FREEMAN[2],
P. NISH[3] AND A.E.O. MALAU-ADULI[1]
[1]University of Tasmania, Private Bag 54 Hobart, TAS 7001, Australia.
[2]TIAR, P.O. Box 3523, Burnie, TAS 7320
[3] Tas Herd, Hadspen, Tasmania

Short title: Lactation profiles of grazing cows

ABSTRACT

Mathematical functions to describe a series of milk test day records have the advantage of minimizing random variation, while simultaneously summarizing the lactation profile. Five empirical functions and two mechanistic models were used to model herd and individual milk yield profiles of multiparous Holstein-Friesian cows on 113, 290 milk yield records (8438 lactations) collected from 1994-2005. The models tested were the incomplete gamma (IG), a modified gamma (MG), an exponential (EXP), a polynomial regression (PR), a mixed log (ML), the bi-compartmental (BC), and Dijkstra (DJ) functions, the latter two being mechanistic models. Each model was fitted using the non-linear (NLIN) function of SAS. Model accuracy was evaluated based on residual mean square (RMS), the magnitude and distribution of residuals, and the correlation between the observed and predicted values. All the models, except MG, did equally well in portraying the lactation profile. Parameter estimates were significant ($P<0.05$), with large serial correlations indicating biased predictions, especially during mid-lactation. Correlations of residuals and observed herd average lactations ranged between -0.13 (MG) to 0.19 (IG), while that between observed and predicted was between 0.76 and 0.99 for the same models. Lactation curves of individual cow milk yields were more varied, exhibited the tendency for a second peak which were not accurately modeled. Mechanistic models performed best with herd data, the PR model fitted overall best, while the MG model fitted the profile least accurately in this study.

Keywords: model, lactation profile.

INTRODUCTION

Mathematical functions such as those previously used to describe a series of milk test day records (Wood, 1967; Cobby & Le Du, 1978; Wilmink, 1987), have the advantage of minimizing random variation while simultaneously summarising the lactation profile into biologically interpretable parameters. Results may be useful in making management and breeding decisions to evaluate the performance of a dairy enterprise.

The functions available to model lactation profiles are many, and include empirical (linear or non-linear), mechanistic, test day and non-parametric models (see review by Beever *et al.* 1991). Despite the better fits based on RMS values obtained from the more complex models, simpler empirical models tend to be preferred by many researchers (Tozer & Huffaker, 1999). Wood's incomplete gamma function is the most commonly used model to fit daily milk yield data, mainly because its three parameters can be related to biological components of the curve. (Saxton *et al.* 2004).

Although the shape of the lactation curve is assumed to be the same for dairy cows, that is an increase to a peak 4-8 weeks into lactation, then a decline until drying off, temporary environmental factors influence yield pattern. For instance, the lactation curve of stall-fed cows differs from that obtained from grazing systems (Garcia & Holmes 2001, Tozer & Huffaker 1999). A robust model must be capable of minimising such distortions, and model the true shape of the profile.

The suitability of different models reported in the literature has been diverse, for instance, Olori et al. (1999) reported that the polynomial model gave the best fit in a farm-based study, while Garcia & Holmes (2001) found no difference in average lactation predicted by both diphasic and linear-based split-plot models. Papajcsik & Bodero (1988) evaluated twenty lactation models and concluded that the Wood's IG model and its derivative [$y=n^b/\cosh(cn)$] gave equally good fits for cows in a sub-tropical environment. In comparison, Val–Arreola *et al.* (2004) fitted five models to data from small scale and intensive systems in Mexico and found that the mechanistic model presented by Dijskra *et al.* (1997) was best.

The objectives of this study were to compare the accuracy of prediction of some of the more commonly used lactation models for pasture-based Holstein-Friesian (HF) cows in a temperate climate, and identify which models best represent the lactation profile of pasture-based cows at either a herd or individual cow level.

MATERIALS AND METHODS

The data used in this study were collated from the Elliot Dairy Research and Demonstration Station (ERDS; North-west Tasmania) and from the state-wide Tasherd databases comprising 428 dairy herds, henceforth referred to as DATA1 and DATA2 respectively. DATA1 consisted of 61,000 milk yield records from 1994-2005, while DATA2 comprised 65,000 milk test-day records from 2000-2005. Dairying in Tasmania is primarily based on perennial ryegrass (*Lolium perenne* L.*)* and white clover (*Trifolium repens*) pastures, with strategic use of grain during periods of pasture deficit. Calving season and breeding programs were similar across data sets

Lactation records with obvious coding errors (e.g. calving date preceding birth date) containing less than seven test day records for a particular lactation, and which were <100d or >350d were excluded from the analyses, but not the entire records in the latter case. No adjustment factors were used. The final data set consisted of 57,722 (1,987 lactations), and 55,555 (6,451 lactations) records for DATA1 and DATA2 respectively. Parities greater than four were pooled. Lactation stage in weeks (WIM) was obtained by dividing the difference between test date and calving date by seven. Summary statistics for parity and herd characteristics are presented in Table 1.

The five empirical and two mechanistic functions used to evaluate average daily milk (kg/d) were:

1.	Incomplete Gamma	$Y(t) = a\,t^{b}e^{-ct}$	(Wood, 1967)
2.	Modified Gamma	$Y(t) = a\,te^{-ct}$	(Jenkins & Ferrel, 1984)
3.	Mixed Log	$Y(t) = a+bt^{1/2}+c\log t$	(Guo & Swalve, 1995)
4.	Exponential	$Y(t) = a+be^{-kt}+ct$	(Wilmink, 1995)
5.	Polynomial	$Y(t) = a+bt+ct^2+d\log t+\varepsilon(\log t)^2$	(Ali & Schaeffer, 1987)
6.	Bicompartmental	$Y(t) = a\,e^{-bt} + de^{-ct}$	(Ferguson & Boston, 1993)
7	Dijkstra	$Y(t) = a\,\exp[b(1-e^{-ct})/c-dt]$	(Dijkstra *et al.*, 1997)

where Y is milk yield (kg/d), at time t (weeks), and a, b, c d and ε are parameters that define the scale and shape of the curve.

Statistical analysis

Data were analysed using the Marquardt's iterative method of the non-linear (NLIN) procedure of SAS (SAS 2005) on lactation stage (WIM) of herd and individual cow's milk yields. Parameter estimates were compared only within models and across data sets as each model gives unique estimates. Evaluation of the models' accuracy was based on its ability to converge, on RMS, the magnitude and distribution of residuals and the correlation between observed and predicted milk yield. The exponential model of Wilmink has a constant parameter (k) which was fixed (Wilmink 1987) in this study, at 0.46, this being the best fitting value for herd mean yield in a preliminary analysis of the data sets, during which the initial values of the NLIN procedures were also determined.

Table 1. Summary statistics for daily milk yields (kg/d)

Data Set	Parity	Mean±SE	Min	Max	CV	Nobs
DATA1	1	15.0±0.04	0.60	41.44	29.44	13, 271
	2	17.3±0.05	1.20	42.72	30.51	10, 568
	3	19.4±0.06	0.90	48.01	29.90	8, 918
	4	20.3±0.07	3.50	49.68	30.05	7, 933
	>4	20.3±0.05	1.40	54.60	31.75	17, 045
DATA2	1	10.5±0.04	1.80	26.90	38.75	9, 463
	2	11.8±0.05	2.0	30.11	39.07	9, 574
	3	12.6±0.05	1.30	44.49	40.63	8, 864
	4	13.0±0.06	1.60	33.90	40.75	8, 656
	>4	13.1±0.04	2.00	40.81	39.09	19, 998

Cows with parity >4 were pooled

RESULTS AND DISCUSSION

HERD:

For DATA1, milk yield (kg) started at 21.1, peaked at 23.4, in week 4 and declined gradually before stopping at 10.9 for DATA1, compared to 12.0, 14.6 (week 4) and 10.4 (DATA2). Similar values have been reported for pasture-based dairy cows (Olori *et al.* 1999, Garcia & Holmes 2001). Parameter estimates in all the empirical models were significant and serially correlated. Estimates of parameter "a" were generally lower in DATA2 for all models being 2.8 and 14.9 for MG and BC models respectively compared to 4.9 and 25.8 for the same models in DATA1. The rate of increase to peak (b) did not differ between the IG model across data sets but the values were higher in DATA1 for EXP, DJ, and BC models while it was higher in DATA2 for the PR model. Higher values of parameter "b" is indicative of increasing milk yield or increasing specific rate of cell proliferation associated with cow genetic merit or improved nutrition (Pollot 2000). The decay parameter "c" as estimated by DJ and BC models was similar but was always higher in DATA1 being 1.10 for both models compared with about 0.74 (DATA2). The specific rate of cell death showed similar trends indicating a faster decline post peak yield for cows in DATA1.

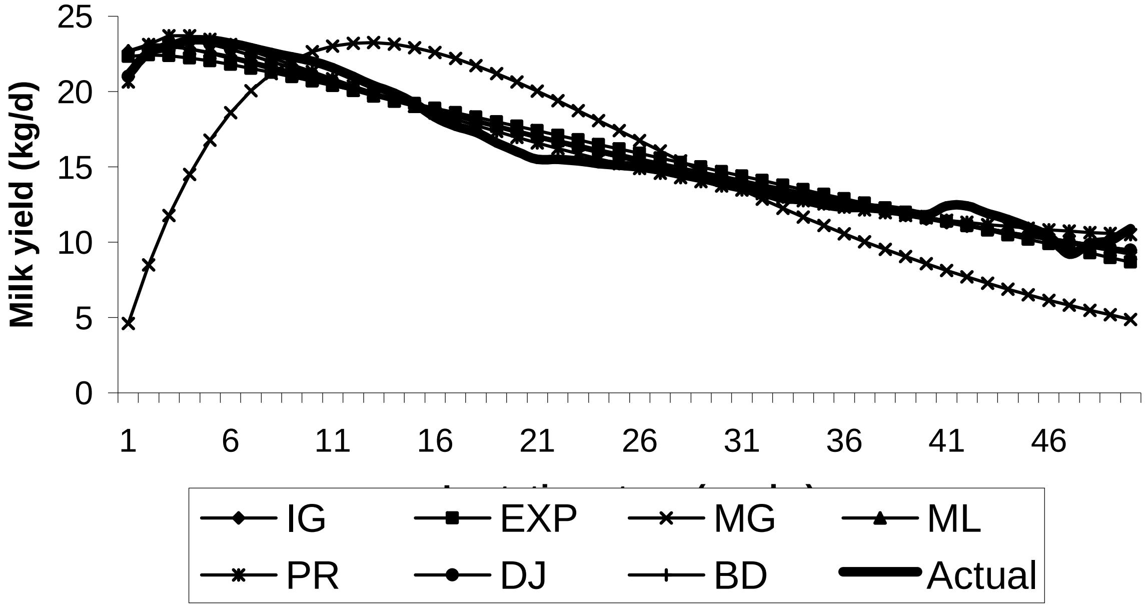

Figure 1. Herd actual and predicted milk yield profile (DATA1) of pasture-based Holstein-Friesian dairy cows fitted to different lactation functions. Lactation functions are: Incomplete Gamma (IG), Exponential (EXP), Mixed Log (ML), Modified Gamma (MG), Polynomial Regression (PR), Dijskra (DJ) and Bi-compartmental (BC).

Predicted initial, peak, mid and final milk yields for the different models with RMS are shown on Table 2. The three parameter models (EXP, IG and ML) over-predicted initial milk yield (kg) by between 0.7 and 1.6. The MG model generally under-predicted initial milk yield by 15 to 17kg. Bias in prediction of lactation around peak, mid and late lactation has been reported in studies by Cobby and Le Du, (1978) and Rowlands *et al.* (1982). The mechanistic and PR models gave the most accurate predictions of initial, peak, mid and final milk yields with very precise week at peak in both data sets. Final yield and week at peak were well predicted by all the models except MG which overestimated peak week by 9 to 11 weeks. Pearson correlations of actual milk yield with predicted and residual values ranged between 0.78 to 0.99 and -0.13 to 0.22, respectively for DATA1, while the corresponding figures for DATA2 were 0.20 to -0.96 and 0.05 to 0.45, respectively. The best models for herd data evaluation were the PR and the mechanistic models, while Wood's IG was the best of the three parameter models. Our results are similar to findings reported by Olori *et al.* 1999, Tozer & Huffaker 1999 and Val-Arreolar *et al.* 2004 respectively but differs from the conclusions of Olori *et al.* 1999 with respect to Wilmink's EXP model.

INDIVIDUAL COWS

All the models were fitted to a random selection of 100 cows from each data set using the NLIN approach as previously described. The goodness of fit was evaluated on iterative convergence and the proportion of individual cows well fitted as indicated by RMS value. The proportion of individual fits corresponding to four RMS classes; 0 to 0.9, 1.0 to 5.0, 5.1 to 15 and >15, corresponding to a very good, good, fair and poor fit, were compared (Table 3). All the models except MG fitted 69% of individual cow's lactation as very good or good, although the mechanistic models did not converge in DATA2. The polynomial model of Ali and Schaeffer still outperformed the rest in goodness of fit. The MG fitted 63% of individual lactation poorly.

All the models gave a uniform but larger variation in RMS for individual cows (compared to the Herd Data), especially where a large deviation in yield or missing data resulted in poor fits. Insufficient milk records in early lactation, the influence of environmental factors and variation between cows can account for poor model fits (Wood 1969). Fluctuation in the milk yield profile of pasture-based cows can also arise from variation in pasture metabolisable energy content due to seasonal variation (Kolver & Muller 1998).

Table 2: Herd actual and predicted initial, peak, mid-lactation and final milk yields (kg/d) of pasture-based HF cows. Residual mean square (RMS) was obtained by fitting average milk yield data to seven lactation functions.

DATA SET	ACTUAL		PREDICTED LACTATION MODELS						
	ITEM		EXP	IG	MG	ML	PR	BC	DJ
DATA1	Initial	21.1	22.3	22.7	4.59	22.7	20.6	21.0	21.0
	Peak	23.4	22.4	22.8	23.2	23.1	23.7	23.4	23.4
	Mid	15.0	19.9	15.5	17.1	15.6	15.0	15.4	15.4
	Final	10.9	10.9	9.34	10.9	10.9	10.9	10.9	10.9
	Peak week	4	2	4	13	2	4	4	4
DATA2	RMS		1.15	0.66	25.2	0.74	0.31	0.47	0.47
	Initial	12.0	12.7	13.2	2.3	12.8	12.1	12.0	12.0
	Peak	14.6	13.9	13.8	15.1	13.9	14.2	14.3	14.5
	Mid	12.4	12.0	12.0	13.7	11.9	11.8	11.9	11.9
	Final	10.5	9.4	9.38	6.3	9.4	9.8	9.5	9.5
	Peak week	4	5	5	17	5	5	4	4
	RMS		0.24	0.26	10.4	0.23	0.19	0.19	0.19

* Lactation functions are: Exponential (EXP), Incomplete Gamma (IG), Modified Gamma (MG), Mixed Log (ML), Polynomial Regression (PR), Dijskra (DJ) and Bi-compartmental (BC)

Table 3. Frequency distribution of lactation profile fits among different classes of residual mean square (RMS), for selected individual pasture-based Holstein-Friesian cows.

		Class of Residual Mean Square (RMS) %					
DATA SET	MODEL	0 – 0.9	1.0 – 5	5.1 – 15	>15	NC	TOTAL
DATA1	EXP	0	70	23	7	-	100
	IG	0	71	23	6	-	100
	MG	0	7	30	63	-	100
	ML	0	70	24	6	-	100
	PR	0	80	14	6	-	100
	BC	0	41	11	7	41	100
	DJ	0	47	14	10	29	100
DATA2	EXP	0	24	56	20	--	100
	IG	0	23	56	21	-	100
	MG	0	8	30	62	-	100
	ML	0	21	58	21	-	100
	PR	1	18	54	27	-	100
	BC	0	0	0	0	100	100
	DJ	0	0	0	0	100	100

NC= No convergence, BC and DJ did not converge on DATA2

CONCLUSION:

Seven models have been evaluated for goodness of fit to pasture-based data. The best overall model for fitting either herd or individual cow data was the PR model, but Wood's IG and Guo and Swalves ML model were the best of the three parameter models. The mechanistic models performed best in fitting herd production. The results underscore the need to use appropriate functions for dairy evaluation and has implications for management in pasture-based systems.

ACKNOWLEDGEMENT:

The authors would like to thank the management of Elliot Dairy Research and Demonstration station, and TasHerd access to the data and Prof. David Ratkowsky for statistical advice.

In: DF Chapman, DA Clark, KL Macmillan, DP Nation (Editors). Dairy Science 2007, Meeting the Challenges for Pasture-Based Dairying', *Proceedings of the 3rd Australasian Dairy Science Symposium*, The University of Melbourne, Victoria, Australia, 3:423-430 (2007)

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231