Exploring Evidence of Mathematical Tasks and Representations in the Drawings of Middle School Students

Vesife Hatisaru [1*]

[1] University of Tasmania, AUSTRALIA

* CORRESPONDENCE: ✉ vesife.hatisaru@utas.edu.au

ABSTRACT

The drive to explore students' experiences in mathematics classrooms remains imperative for mathematics education research in order to better understand how effective teaching and learning classroom practices lead to desirable learning outcomes. As part of a larger research project exploring a group of 120 Turkish middle school students' (grades 6 to 8, aged 11 to 14) perceptions of their mathematics classroom experiences, this article presents an analysis of the nature of mathematical tasks and the forms of mathematical representations depicted in students' drawings. An analysis of the data obtained from the students' drawing task (Draw a Mathematics Classroom Test) revealed little to no variety in students' classroom experiences in relation to the types of mathematical tasks and mathematical representations. The most common mathematical tasks were found to be tasks that focus on procedural skills, while the most common way students represented the mathematics was through symbolic representations. None of the student drawings involved physical or contextual representations. Findings raise concerns about whether Turkish students are well prepared for the demands of the 21st century.

Keywords: classroom experiences, drawings, mathematical tasks, mathematical representations, middle school students

INTRODUCTION

The mathematics learning that effective teaching practices can inspire and develop includes the achievement of five intertwined strands, which together constitute mathematical proficiency: *conceptual understanding* (comprehension of mathematical concepts and operations), *procedural fluency* (skill in carrying out procedures accurately and efficiently), *strategic competence* (ability to formulate and solve mathematical problems), *adaptive reasoning* (capacity for logical thought, reflection and justification), and *productive disposition* (seeing mathematics as sensible, useful and worthwhile) (see Kilpatrick, Swafford, & Findell, 2001). In alignment of the first four of these strands, *understanding*, *fluency*, *problem solving* and *reasoning* in mathematics have been accepted to represent the bases for mathematical proficiency (Australian Curriculum Assessment and Reporting Authority [ACARA], 2016). These bases for mathematical proficiency have been variously described as the practices that students engage with and demonstrate that they can do as they learn and use the mathematical content (ACARA, 2016; National Council of Teachers of Mathematics [NCTM], 2014). With context related factors (e.g., the school, home, wider education system) in the lesson implementation in mind (Kilpatrick et al., 2001), the nature of classroom mathematics teaching impacts the development of mathematical proficiency in students (Hatisaru, 2017; Hiebert & Grouws, 2007; NCTM, 2000).

The teaching that fosters mathematical proficiency can take a variety of forms. The relevant literature reveals that specific teaching practices appear to be common for desirable learning outcomes. These practices

include: implementing real and relevant tasks (Bobis, Anderson, Martin, & Way, 2011) that promote problem solving (Anthony & Walshaw, 2009; NCTM, 2014); using a variety of questioning including open, high-level ones (Bobis et al., 2011; Swan, 2005); engaging students with learning tasks by using manipulatives representing mathematical ideas (Anthony & Walshaw, 2009; Kilpatrick et al., 2001); using and connecting mathematical representations (Anthony & Walshaw, 2009; NCTM, 2014); and building procedural fluency from conceptual understanding (NCTM, 2014). Like teachers from around the world, teachers of mathematics in Turkey are encouraged to embrace research-informed practices that have been found to be successful for mathematics learning (Ministry of National Education [MoNE], 2018).

Results from international comparative studies, which intend to regularly assess proficiencies of school students in mathematics and science, such as the Trends in International Mathematics and Science Study (TIMSS), address the low proficiency level in middle school students in Turkey against international benchmarks (Mullis, Martin, Foy, & Hooper, 2016). Also, students' participation in tertiary mathematics courses (e.g., Nesin, 2014) and interest in entering careers in science (e.g., Narayan, Park, Peker, & Suh, 2013) have been in decline. Given that students often exhibit poor learning outcomes in mathematics, my motivation has been driven by an attempt to identify possible challenges students face in mathematics classrooms which might contribute to these poor outcomes. In this paper, I report on research which is part of a larger study, in which I investigated Turkish middle school students' perceived experiences of their mathematics classroom (e.g., listening to lectures, working with peers, using mathematical tools) through examining their visual and verbal descriptions (Hatisaru, submitted). Specifically, I present the analysis of the mathematical practices represented in their diagrams and text, in relation to the following research questions: *What is the nature of mathematical tasks in students' drawings? What forms of mathematical representations are used? Are there any patterns in these depictions?*

There is consistent evidence that students' perceptions of classroom learning environments are associated with their learning outcomes (Fraser, 2014). Large-scale assessments such as TIMSS and the Program for International Student Assessment (PISA) have identified important aspects of the perceptions of students in regards their classroom experiences. In the absence of observational and/or interview data those studies suffer from findings limited to responses gleaned from questionnaire items (Vieluf, Kaplan, Klieme, & Bayer, 2012). The statements used in questionnaires are not necessarily understood by younger students in the way researchers intended (Bragg, 2007). More detailed information would be beneficial in exploring perceived mathematical experiences in the classroom and may help to alleviate some of the limitations in the existing literature. More importantly, without evidence addressing these matters, it is difficult to assess the claims about students' poor performance in mathematics and/or negative images about mathematics, and the possible sources of them.

Below, I critique the literature before discussing the context of the study. I then present the research instrument, Draw a Mathematics Classroom Test, followed by the research design and data analysis. Finally, I present the results of analysis, draw preliminary conclusions about the mathematical tasks and forms of representations used in the classroom seen through the students' eyes, and provide implications for research and practice.

THEORETICAL FRAMEWORKS

Literature from several areas has informed this research: the image of mathematics; characteristics of mathematical tasks; representations in mathematics; and using drawings to study the image of mathematics.

The Image of Mathematics

The literature has shown that there is no universal definition in the literature of 'the image of mathematics' construct as seen in the following examples. Brown (1992) defined the image of mathematics as the feelings, expectations, experiences and confidence individuals hold about mathematics. Synthesizing the literature on conceptions of image, Sam and Ernest (2000) conceptualised the construct as "a mental representation or view of mathematics, presumably constructed as a result of social experiences, mediated through school, parents, peers or mass media" (p. 195). Wilson (2011) proposed an operational construct to define the factors that might influence individuals' engagement in mathematical activity which coincides with the image of mathematics construct. Wilson used the term 'disposition' composing of beliefs, emotions, motivation, and needs. Combining the definitions of Wilson (2011) and Sam (1999), and other research in the affective domain, Lane, Stynes, and O'Donoghue (2014) defined the image of mathematics as "a mental representation or view of mathematics,

Table 1. Components of the image of mathematics

Sam and Ernest (2000)	Stated attitudes
	Feelings or choice of emotive descriptors
	Descriptions or metaphors for learning mathematics
	Beliefs about the nature of mathematics
	Views about mathematicians and their work
	Beliefs about mathematicians' ways of knowing and warranty of mathematical knowledge
	Descriptions or metaphors for mathematics learning
	Aims for school mathematics
	Memories of best or worst mathematics lessons
	Beliefs about mathematical ability
	Beliefs about sex differences in mathematical ability
Wilson (2011)	Beliefs, values, or identities
	Affect or emotions
	Behavioral intent or motivation
	Needs
Lane et al. (2014)	The affective domain (attitudes, emotions, and self-concepts relating to mathematics and mathematics learning experiences)
	The cognitive domain (beliefs relating to mathematics and mathematics learning experiences)
	The conative domain (motivation relating to mathematics learning)

presumably constructed as a result of past experiences, mediated through school, parents, peers or society" (p. 881). All these authors conceptualized the image of mathematics as a multifaceted construct composed of several aspects that are summarized in **Table 1**. Both the nature of perceived mathematical tasks and forms of mathematical representations depicted in the mathematics classrooms underpin students' perceptions of their mathematics teaching and learning experiences and their image of mathematics as represented in drawings.

The image of mathematics has been found to influence student performance (Wong, Marton, Wong, & Lam, 2002), interest in mathematics (Latterell & Wilson, 2012), and attitudes about mathematics (Picker & Berry, 2001). Presumably, due to those influences, the image of mathematics ranks highly among student affective outcomes identified by both standard documents (Organisation for Economic Co-operation and Development [OECD], 2016) and scholars (Ernest, 2010; Lane et al., 2014). To date, certain components of this construct (e.g., attitudes, feelings, beliefs relating to mathematics) have been widely investigated (e.g., Aguilar, Rosas, Zavaleta, & Romo-Vázquez, 2016; Hatisaru, 2019; Rock & Show, 2000). The research in this area needs more information on the perceptions of students relating to their mathematics learning experiences. As mentioned before, large-scale assessments such as TIMSS and PISA identify various aspects of classroom climate through the eyes of students, but these surveys have been unable to identify types of teaching practices and limited to responses to questionnaire items (Vieluf et al., 2012). The pictures produced by the students contain valuable insight into mathematical practices in the classroom and can inform policy making efforts aimed at optimising opportunities for student mathematical learning and leading to desirable outcomes.

Mathematical Tasks

For decades, data have been collected about different aspects of teaching, including teacher characteristics, skills and practices (Blazar, 2015) and associations between teachers' use of specific teaching strategies and student achievement (O'Dwyer, Wang, & Shields, 2015). One of the major findings of this research is that the nature of the mathematical tasks offered to students in the classroom does matter (Stein & Smith, 2000), for both mathematical understanding and enjoyment and relevance in the learning of mathematics (Anthony & Walshaw, 2009; Swan, 2005).

A task is "a segment of classroom activity that is devoted to the development of a particular mathematical idea" (Stein & Smith, 2011, p. 9) and can range from a set of routine exercises to a complex and challenging problem (NCTM, 2014). Presenting students with a range of sufficiently rich tasks that can provide them with opportunities to develop their conceptual understanding of key ideas, reasoning, problem-solving, communication skills, as well as procedural skills (Sullivan, 2011) is an expectation of curricula worldwide (e.g., ACARA, 2016; MoNE, 2018; NCTM, 2014). Students can discover or develop their mathematical capacity through engaging with mathematical tasks, and to become skilled learners of mathematics, they should be presented with quality tasks (Anthony & Walshaw, 2009).

Table 2. Examples of tasks at each of the four levels of cognitive demand (Smith & Stein, 1998; Stein & Smith, 2011)

Cognitive demand	Example
Lower-level (memorization)	What is the rule for multiplying fractions?
	What are the decimal and percent equivalents for the fractions $\frac{1}{2}$?
Lower-level (procedures without connection)	Multiply $\frac{2}{3} \times \frac{3}{4}$; $\frac{5}{6} \times \frac{7}{8}$
	Convert the fraction $\frac{3}{8}$ to a decimal and a percent.
Higher-level (procedures with connections)	Find $\frac{1}{6}$ of $\frac{1}{2}$. Use pattern blocks. Draw your answer and explain your solution.
Higher-level (doing mathematics)	Create a real-world situation for the following problem: $\frac{2}{3} \times \frac{3}{4}$. Solve the problem you have created without using the rule and explain your solution.

The literature provides different elaborations in relation to the quality or nature of mathematical tasks. Based on a wide range of research, Anthony and Walshaw (2009) argued that students should be presented with tasks that allow for original thinking about mathematical concepts or relationships, and not always tasks that involve practicing the algorithms they have been taught. The authors found open and contextual tasks, for example creating a schedule for producing a family meal, which require students to practise mathematics (e.g., investigating, creating, reasoning, communication), worthwhile for developing these practices in students.

Drawing on comprehensive research on associations between mathematical tasks and student learning, Stein and Smith developed a taxonomy of mathematical tasks based on the level of cognitive demands that a task requires students to solve it (see Smith & Stein, 1998; Stein & Smith, 2011). They defined cognitive demand as the kind of thinking and effort needed to solve a task. The taxonomy defines the kind and level of thinking required to solve mathematical tasks from *Lower-level demands* to *Higher-level demands*. According to Smith and Stein (1998), *Lower-level demand (memorization)* tasks involve committing facts, rules, formulas or definitions to memory. They are not ambiguous and usually do not require the use of procedures as no procedure exists. *Lower-level demand (procedures without connection)* tasks are algorithmic and require the use of procedures; therefore, little ambiguity exists about what needs to be done and how to do it. They require limited cognitive demand for successful completion. *Higher-level demand (procedures with connections)* tasks require some degree of cognitive effort and suggest explicit or implicit pathways to follow. They usually incorporate different representations, including such things as symbols, visual diagrams and verbal statements, and require students to make connections among different representations to develop meaning. *Higher-level demand (doing mathematics)* tasks are more complex and involve considerable cognitive effort. They require non-algorithmic thinking and students need to explore and understand the nature of mathematical concepts or processes. These types of tasks require students to access relevant knowledge and use it appropriately while working through the task. **Table 2** presents examples of tasks at each of the four levels of cognitive demand presented by the authors.

Sullivan (2011) also categorised mathematics tasks according to the learning outcomes they can achieve. He found that the most commonly used tasks in many mathematics classrooms are those that give students opportunities to practice procedures in order to become fluent in them, what Kilpatrick et al. (2001) called *procedural fluency* (see **Table 3** the first row, for an example). Sullivan argued that although practicing the procedures is needed, it is essential that students are offered tasks that are not only repetitious practices of demonstrated algorithms. Sullivan identified three other groups of mathematical tasks: tasks requiring the use of models or representations; tasks drawing from realistic contexts; and open-ended tasks (see **Table 3**). Tasks requiring the use of models or representations intend to make abstract mathematical concepts or ideas more concrete through the using of, for example, physical, visual or symbolic representations. Tasks involving realistic contexts provide both a stimulus for learning, and can improve students' ability to formulate and solve mathematical problems (*strategic competence*) and develop their capacity for logical thinking and justification (*adaptive reasoning*) (Kilpatrick et al., 2001). Finally, open-ended tasks that enable the use of different solution strategies and can result in different solutions provide opportunities for rich classroom discussions and allow and encourage students to explain their methods and underlying thinking (see **Table 3**, for examples of tasks at each of the four groups and Sullivan, 2011, for more details).

Table 3. Examples of tasks at each of the four groups Sullivan (2011) suggested

The nature	Example
Procedural	Can you solve $7x + 4 = 5x + 8$?
Representational	Giving students cards depicting the same mathematical idea or concept (e.g., polyhedron) in different ways (e.g., verbal, visual, pictorial descriptions) and asking them to match the cards to enable them to draw links between the different representations of the same concept and to develop new mental images for it.
Contextual	If one pre-paid card for downloading music offers 16 songs for $24, and another offers 12 songs for $20, which is the better buy?
Open-ended tasks	On squared paper, draw as many different parallelograms as you can with an area of 12 square units.

Together, the reviewed literature has shown that students need to be presented a range of and sufficiently rich tasks that can provide them opportunities for conceptual understanding of key ideas, reasoning, problem solving, and communication; as well as for development of procedural skills.

Mathematical Representations

In the context of mathematics teaching and learning, a representation is a form of an idea or concept that allows us to interpret, communicate, discuss, and/or manipulate the idea or concept with others (Goldin, 2014; Kilpatrick et al., 2001; Tripathi, 2008). To this end, mathematical representations are visible productions that embody mathematical ideas and include diagrams, graphs, number lines, concrete objects, or mathematical formulae, expressions and equations (Goldin, 2014; NCTM, 2014). The nature of mathematics is abstract, and mathematical ideas can only be accessed through the representations of those ideas (Cramer, 2003; Kilpatrick et al., 2001; Pape & Tchoshanov, 2001) or tools (Anthony & Walshaw, 2009; NCTM, 2014). The use of representations and tools in mathematics classrooms is therefore essential to learning as they can assist in creating, shaping and mediating ideas (Goldin, 2004; NCTM, 2014; Pape & Tchoshanov, 2001).

Lesh, Post and Behr (1987) identified five distinct types of representation systems that could be used in the teaching of mathematical ideas and that could help students develop deeper understanding of them: *visual, symbolic, verbal, contextual,* and *physical* representations. As elaborated by Johnson (2015), visual representations refer to anything made by hand or generated by computer that represent concrete objects such as a graph, chart, tallies or table. Symbolic representations include numbers, formulae, geometric concepts, and numerical or algebraic expressions. Verbal representations incorporate the specialised language required of mathematical domains (e.g., fractions, probability, geometry). Contextual representations refer to situations happening in the real world (e.g., using money in shopping), while physical representations include concrete objects or manipulatives (e.g., base ten blocks; protractors; geoboards) that are designed to give students opportunities to learn mathematical concepts by manipulating them.

The same mathematical idea could be represented in different representational forms. For example, the concept of a line can be: written as an equation (algebraic); described in words as a collection of points placed side by side (verbal); depicted as a line on a board (visual) or the horizontal value in a pair of coordinates (graphical); or modelled as people or objects standing side by side (concrete). Nevertheless, each representation stands for some aspects of a line even though other aspects may be missed (Tripathi, 2008). According to Lesh et al. (1987), and stressed by Tripathi (2008), each distinct type of representation is important. Yet, the translation within and/or among representations is also important and a crucial element of what it means to understand a mathematical idea or concept. The *translation within representations* occurs within a distinct type of representation system (e.g., visual) and means translating, for instance, a visual model to an array or an area model. While the *translation between representations* occurs when translating from one distinct type of representation system (e.g., visual) to another (e.g., symbolic).

Being able to use different forms of representations is an important component of student understanding in mathematics subjects and in investigating and communicating real-world issues (Goldin, 2004; Pape & Tchoshanov, 2001). When students encounter and use different representations for the same concept, their understanding of the concept is enhanced (Kilpatrick et al., 2001; Pape & Tchoshanov, 2001), and they develop new or richer mental images for the concept (Swan, 2005).

USING DRAWINGS TO STUDY THE IMAGE OF MATHEMATICS

The research reported here and elsewhere (e.g., Hatisaru, 2019a) used data derived from students' drawings about their image of mathematics. The scientific interest in children's drawings arose in the 1900s, and drawings have been used for psychology, anthropology and ethnology (e.g., Goodenough, 1926) as well as for education (Chambers, 1983). The research capturing students' perceptions of science education through drawings arose after the seminal work of Mead and Métraux (1957) examining the perceptions students held about scientists. Through the years, the Draw a Scientist Test (DAST) (Chambers, 1983) was patterned from Goodenough's (1926) the Draw a Person Test. For facilitating ease of assessment, Finson, Beaver, and Crammond (1995) developed the Draw a Scientist Test Checklist (DAST-C). Later years, Thomas, Pedersen, and Finson (2001) modified the DAST-C to create the Draw a Science Teacher Test Checklist (DASTT-C) and used it to document the preservice elementary teachers' knowledge and beliefs about elementary science teaching methods.

Researchers in mathematics education, such as Picker and Berry (2000), refocused the DAST to enable students to draw a mathematician on a blank sheet of paper and to describe the images reflected in students' drawings of mathematicians. The instrument is entitled the Draw a Mathematician Test (DAMT) and includes a section for students to describe elements of the drawings, which Picker and Berry (2000) assumed would enable students to provide more information about their beliefs. The use of drawing tasks with accompanying text, later, have been found to add rigor to the instrument as the information provided in the writing reduces the subjectivity effect in coding the drawings (Murphy, Delli, & Edwards, 2004).

Over the years, the use of drawings in education as a measure of students' conceptions of teaching and learning school subjects (e.g., mathematics) (Johansson & Sumpter, 2010) have been found to be valid (e.g., Gulek, 1999; Laine, Ahtee, & Näveri, 2020; Losh, Wilke, & Pop, 2008), reliable (e.g., Johansson & Sumpter, 2010; Remesal, 2009) and useful (e.g., Harris, Harnett, & Brown, 2009), as well as a cost-effective alternative to classroom observations (Haney, Russel, & Bebell, 2004). As a research method, it has offered more opportunities to students, especially at young ages, to express their core opinions about mathematics than responses to questionnaire items (Stiles, Adkisson, Sebben, & Tamashiro, 2008). Through this method, students can draw freely about their experiences of mathematics teaching and learning (Kearney & Hyle, 2004).

For decades now, drawings have been widely used to elicit data from students relating to their views about mathematics (Rock & Show, 2000; Ucar, Piskin, Akkas, & Tasci, 2010), mathematicians (Aguilar et al., 2016; Hatisaru, 2019a; Picker & Berry, 2001), mathematics teaching (Author, 2019b), and mathematics education with a focus on motivation (Johansson & Sumpter, 2010). Drawings have also been used for exploring students' perceptions of assessment practices in mathematics classrooms (Remesal, 2009), the kind of emotional atmosphere (Laine, Näveri, Ahtee, Hannula, & Pehkonen, 2013) and types of work experienced in mathematics lessons (Pehkonen, Ahtee, & Laine, 2016), and the teacher actions factor on the emotional atmosphere of mathematics classrooms (Laine et al., 2020).

In the research reported in this paper, I used 'drawings' as a measure of the perceptions of middle school students' (aged 11 to 14) experiences in mathematics classrooms using the Draw a Mathematics Classroom Test, adapted from the 'draw a scientist' technique. Student drawings have not been used to find out how students perceive the types of mathematical tasks and forms of representations used in their mathematics classes. In this paper, I provide additional evidence with respect to students' perceptions of these two key aspects of mathematics teaching and learning practices.

METHOD

Context

In Turkey, elementary education lasts for eight years (grades 1 to 8; ages 5 to 13), four years primary and four years middle school education. Teaching at schools is regulated by the national curriculum. Mathematics is taught as a mandatory and major subject during the primary and middle years of school, and it is part of the high school entrance, and in later years university entrance exams, which students sit at the completion of the middle and high school respectively. Mathematical questions make up 22% of the questions for high school entrance exam and 33% of the questions for the university entrance exam (European Schoolnet, 2018). In society, a high status is attached to having a university degree. Both high school and university entrance exams are therefore highly competitive. Parents are excessively involved in education (Altinyelken & Sozeri,

2017). Students generally give much importance to mathematics mostly because mathematics helps in the exams that determine which high school or university a student goes (Hatisaru, 2020).

All teachers must hold at least a bachelor's degree in the subject they will be teaching. Initial teacher education is provided by faculties of education at universities and lasts for four years. The content and structure of the programs are decided by the Higher Education Council with some flexibility given the faculties in deciding the courses offered (Eurydice, 2019a). The programs aim to develop future teachers' general pedagogy, content, and classroom practice of the subject they will teach. The current lower secondary mathematics programs train teachers in three core aspects: general culture (15-20%) (e.g. Foreign Language, and Information Technologies); pedagogic formation (25-30%) (e.g. Sociology of Education, Ethics in Education, and School Experience); and content teaching (50-60%). Content teaching courses are structured by the teaching of the five learning areas in the curriculum such as, Teaching Numbers and Operations and Teaching Algebra (Eurydice, 2019b). In-service training is provided by the Directorate for Teacher Education and Development, in cooperation with universities. The training programs generally focus on general areas such as computer, foreign language, personal development, and knowledge polishing (Eurydice, 2019b). Some mathematics teachers found these programs limited in updating and revising their professional knowledge (Hatisaru, 2018). The need for improving the quality of the training programs and providing varied training activities for teachers based on their needs has been posited in the 2017-2023 Teacher Strategy Document published by the Directorate (2017).

Educational policies related to mathematics competences have been promoted since the revision of school mathematics curriculum in 2004. The revised mathematics curriculum is "based on constructivist principles, student-centeredness, and a departure from lecturing, moving instead toward understanding, exploring, and conceptualizing the essence of mathematical ideas" (Ozdemir, Gonen, Polat, & Ari, 2015, p. 4). The specific goals of the curriculum for students include: developing and using mathematical literacy skills effectively; understanding mathematical concepts and using them in daily life; expressing their reasoning in problem solving processes; and representing the concepts in different representational forms (MoNE, 2018, p. 8, translation by the author).

In the curriculum five learning areas are presented, together achievement standards for each of them: Numbers and Operations; Algebra; Geometry and Measurement; Data Analysis; and Probability. With a level of flexibility available, the teachers are provided advices about the implementation of the curriculum such as, teachers should: use manipulatives (e.g., number cards, base ten block, fraction tiles) in introducing new concepts and in assessments, when possible; make students to express their thinking orally and communicate them both individually and in groups; and pose questions giving students opportunities for explaining and reinforcing their thinking when learning mathematical concepts (MoNE, 2018, p. 13-14, translation by the author). Within the current curriculum context, the reality of classrooms often can be different (e.g., Hatisaru, 2019b). Student depictions in this study could contain an insight into the classroom mathematical practices.

Data Collection and Participants

Qualitative data were collected from middle school students using the Draw a Mathematics Classroom Test (see **Appendix A**), adapted from Thomas et al.'s (2001) Draw a Science Teacher Test (DASTT) and Gulek's (1999) work on using drawings to examine the educational ecology of classrooms. The test provides a rectangular area in which participant students are asked to draw, through the use of a prompt inspired by Gulek (1999): "*Think about teachers of mathematics and the kinds of things you do in mathematics classrooms. Draw a picture of your teacher teaching and yourself learning*". At the bottom of the sheet, the following prompts is given to get students to describe their drawing: "*Look back at the drawing and explain your drawing so that anyone looking at it could understand what your drawing means. What is the teacher doing? What are the students doing? What materials and tools are they using?*" (Picker & Berry, 2001; Thomas et al., 2001). This written narrative component contributes to gaining a deeper understanding of what students have drawn and confirming the interpretations of input in drawings.

A convenience sample of 400, grades 6 to 8 students (aged 11 to 14 years) enrolled in three different middle schools in the 2018-2019 academic year, located in Ankara, participated in data collection, under the auspices of school principals. The instrument was implemented in Turkish, at the beginning of the school year, by counseling teachers and in time set aside for school counseling, as that was convenient for the schools and minimised disruption. Students took the task individually and were not given extra drawing materials. It took about thirty minutes to complete the task.

In data analysis, I employed priori thematic saturation which refers to the degree to which pre-determined codes or categories being sufficiently replete with instances of data (Saunders et al., 2017). As I began to see the same student depictions and descriptions repeatedly, after the 120th drawing, I found that the categories were saturated and terminated the coding of the remaining drawings (Hatisaru, submitted). Of these 120 responses, 97 (approximately 81%) included depictions or text of the mathematical work (representations, formulas, questions, equations or expressions) engaged in by the teacher and students. These responses provided data pertaining to the characteristics of mathematical tasks and forms of representations used in the classroom from the view point of the participant students. Male (n=49) and female (n=47) students were almost equally represented across this sample, while the number of grade 7 (n=33) and grade 8 (n=52) students were greater than the number of grade 6 (n=12) students.

Data Analysis

To analyse the drawings and associated written descriptions, I used deductive content analysis (Elo & Kyngäs, 2007). I transcribed student texts and pictures (mathematical work in the picture, mostly on the whiteboard) documented using excel spreadsheets. Next, I translated the texts into English. As much of the references to the mathematical work were found in the student drawings, the results were predominantly drawn from the written mathematical work in the pictures. Participant students were designated by codes (e.g., S1, S2 and so on).

Specifically, I grouped the mathematical work in student responses into five learning areas in the curriculum (e.g., Algebra, Numbers and Operations) (see **Table 5** in Results). Few drawings included scribbles; the content area was not known. These were coded as 'Not known'. To identify the nature of mathematical tasks in those learning areas, I analysed the data according to the Sullivan's (2011) categorisation of mathematics tasks: procedural; representational; contextual; or open-ended tasks (see **Table 3** in Literature Review). In several pictures, nevertheless, the written work incorporated either the four basic mathematical operations (e.g., "$2 \times 2 = 4$" S30, grade 8, boy) or a numerical/algebraic equation/expression (e.g., "$2^4 = 16$" S36, grade 8, girl; "$(2 + 3) \times 2$" S76, grade 6, boy; and "$4 + (2x + 2)$" S52, grade 8, boy). In a few, the creator of the picture depicted scribbles on the board and described the mathematical work as performing exercises. As these were basic mathematical work descriptions, I created a new category that emerged from the data: 'Basics'. This category composed of six groups: 'Basic operations'; 'Numerical equations'; 'Algebraic expressions'; 'Performing exercises'; 'Algebraic equations'; and 'Numerical expressions' (see **Table 6** in Results). In other depictions, the mathematical work was assessed according to Sullivan's (2011) classification of mathematics tasks (see **Table 7** in Results). To identify the forms of representations, I used Lesh et al.'s (1987) five distinct types of representation systems that mathematical tasks can entail: visual; symbolic; verbal; physical; and contextual. I also noted the tools, both standard classroom teaching materials such as whiteboard, books and if any, manipulatives, or technological tools appeared in the drawings.

To illustrate the coding, in **Figures 1** and **2**, I present examples of two students' drawings and their descriptions that represent the nature of mathematical tasks and forms of representations used in the perceived mathematics classrooms. My judgements of the responses are presented in **Table 4**.
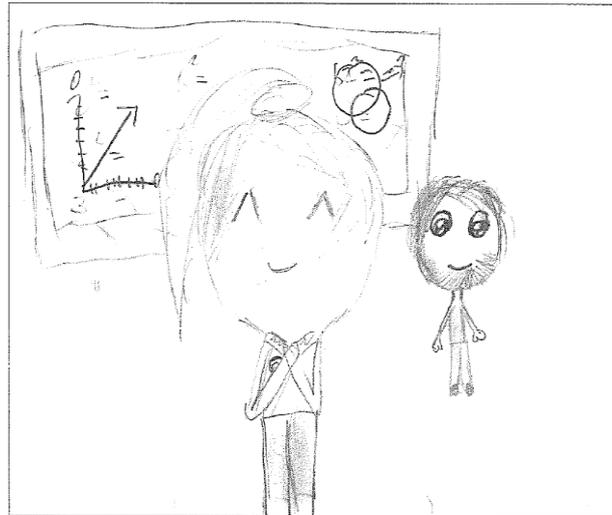
In this picture Teacher Zed [pseudonym] has written something on the board and he is behind me. He is teaching us.
Board marker and eraser, book, pencil. (S81, grade 6, girl)

Figure 2. S81's drawing and description of mathematics classroom

Table 4. Assessments of the student responses shown in **Figures 1** and **2**

Mathematical work	Figure 1	Figure 2
Learning area	Algebra	Numbers and Operations (Sets) Algebra (Linear equations)
Nature of the task	Procedural	Basics
Forms of the representation	Symbolic (algebraic equations)	Visual (graph, set diagram)
Tools	Ruler, Board marker	Board marker and eraser, Book, Pencil
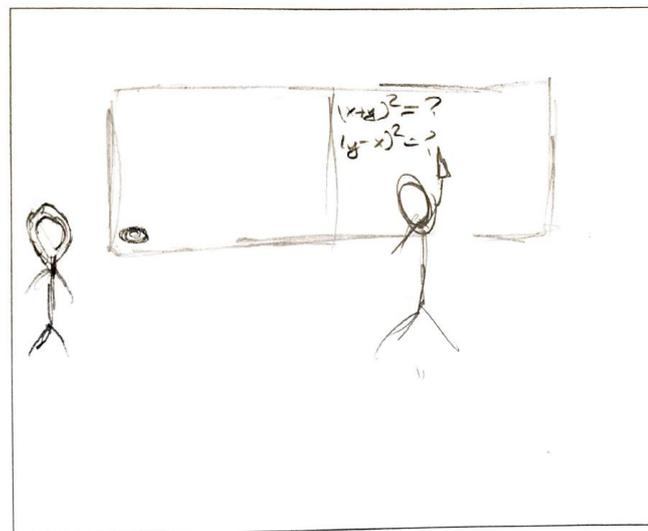
In math class the teacher is asking us to the board and to solve questions. S/he is helping who can't solve. While teaching, the teacher is using ruler, colorful board markers. (S114, grade 8, boy)

Figure 1. S114's drawing and description of mathematics classroom

RESULTS

In this section, I describe a comprehensive analysis of student classroom drawings, presenting examples from their pictures and texts.

Table 5. The written mathematical work in student drawings corresponded to the learning areas

Numbers and Operations	Algebra	Geometry	Probability	Not known
64 (66%)	26 (26.8%)	1 (1%)	1 (1%)	7 (7.2%)

Table 6. The mathematical work depicted in student drawings classified as 'Basics'

| The task	Group	Frequency (%)	Example
Basics	Basic operations	15 (15.5%)	$2 + 2 = 4$ (S3, grade 8, girl) $2 \times 2 = 4$ (S77, grade 6, boy)
	Numerical equations	14(14.4%)	$2.2.2 = 2^3 = 8$ (S24, grade 8, boy) $\sqrt{121} = 11$ (S56, grade 8, boy) $5^{-3} = \frac{1}{125}$ $6^{-3} = \frac{1}{243}$ (S57, grade 8, boy)
	Algebraic expressions	9(9.3%)	a^b; a^{-b} (S31, grade 8, girl) $2x^2.3xy$ (S111, grade 8, boy) $2(3x + 7) + 3(4x + 9)$ (S97, grade 7, girl) $2x + 7$ (S94, grade 7, girl)
	Algebraic equations	6(6.2%)	$2x + 3x = 5x$ (S112, grade 8, girl) $x + 2 = 0$ (S73, grade 7, boy)
	Numerical expressions	6(6.2%)	4^{-5}; 2.2; 2^{-3} (S50, grade 8, boy) $1,81\overline{11} \dots 1,\overline{9}$ (S92, grade 7, girl)
	Performing exercises	7(7.2%)	The teacher has given question. Students are solving question. (S58, grade 8, girl) The teacher comes, writes a question on the board, then asks a student to solve the question. (S49, grade 8, girl)

Table 7. The mathematical tasks depicted in student drawings classified as 'Procedural'

| The task	Frequency (%)	Example
Procedural	37(38.1%)	$\sqrt{100} = ?$ (S2, grade 8, girl) $2^{-2} = ?$ (S23, grade 8, boy) What is the square of $\frac{7}{9}$? (S71, grade 7, boy) $\sqrt{4 + x} = ?$ (S96, grade 7, girl) $2^x = 3, 8^x = ?$ (S14, grade 8, boy) $2^3 = ?$ $4^3 = ?$ $\frac{2^3}{4^2} = ?$ (S34, grade 8, girl) $\frac{2}{3} + \frac{1}{3} = ?$ $2,\overline{5} = ?$ (S91, grade 7, girl)

Mathematical Tasks

Table 5 shows the frequency of references to written mathematical work in the pictures corresponding to the learning areas for the whole sample. Many of the students' (*n*=64, 66%) written mathematical work in the pictures corresponded with the Numbers and Operations learning area, while others (*n*=26, 26.8%) aligned with Algebra. One each of two pictures included written work related to the Geometry and Probability content areas, while no images included Measurement or Data Analysis written work. **Tables 6** and **7** present the perceived nature of the mathematical tasks in these learning areas that the class was engaged in. Two student responses involved multiple learning areas and were coded in more than one group (e.g., across the Algebra and Geometry).

As outlined in **Table 6**, some pictures depicted work using either the four basic mathematical operations (*n*=15, 15.5%) or a numerical/algebraic equation/expression (*n*=35, 36.1%) (e.g., see **Figure 3**). The final group of responses (*n*=7, 7.2%) indicated that the mathematical work was performing questions.
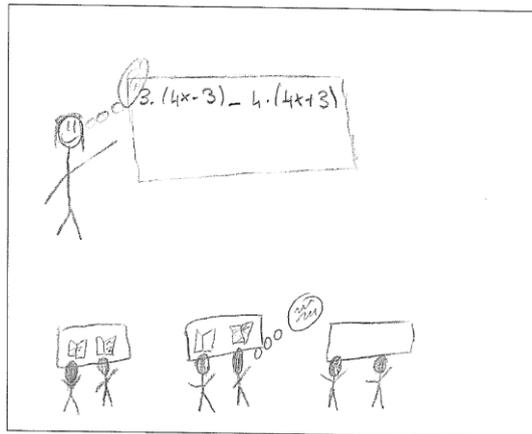
While the teacher is lecturing, we are listening to him nicely and answering. (S86, grade 7, girl)

Figure 3. S86's drawing and description of mathematics classroom
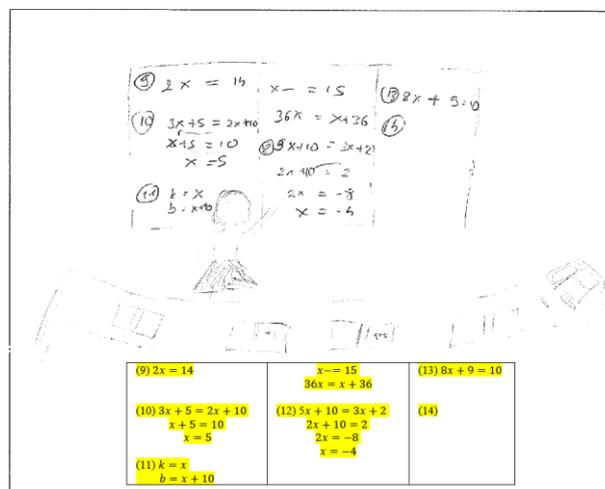
When the teacher solves the homework questions that students couldn't do. Some of the students listen to her, but some are not. (S61, grade 7, girl)

**Figure 4.** S61's drawing and description of mathematics classroom

In this 'Basics' category, the given context was such that the focus of the perceived tasks could be determined to be procedural fluency, with students shown to be using procedures and algorithms to reproduce previously learned facts. To illustrate, in describing his picture, S115 (grade 8, boy) stated: "*The teacher is solving questions at the smart board. Always. All the time, always, until death [always]*". He depicted: "$\ldots + \ldots = \ldots \ldots - \ldots = \ldots \ldots + \ldots = \ldots$" and wrote: "*[The teacher:] Let's perform this question.*" His visual and the written text described a classroom context in which usually involves lots of routine exercises were performed. Similarly, S86 (grade 7, girl) depicted an algebraic expression on the board and used these words in describing her picture: "*While the teacher is lecturing, we are listening to him nicely and answering*", referencing possibly a mathematical task that is solving or simplifying an algebraic equation (see **Figure 3**). Several other students (n=7, 7.2%) scribbled on the board and described the mathematical work as performing exercises: "*After teaching the subject at the beginning and solving a few examples, our teacher is giving us questions and [we are] answering the questions*" (S26, grade 8, boy).

In the remaining responses, the mathematical tasks depicted focussed on mathematical procedures mostly in Algebra (**Table 7**). Within this group, most of the tasks consisted of performing standard algorithms with fractions, square roots, exponentials (e.g., see responses of S2 and S23 in **Table 7**) or solving algebraic expressions (e.g., see **Figure 1** and **Figure 4**). The pathway in these tasks is implied as they are routine exercises. They neither seem to encourage the use of different strategies such as drawing a diagram, making a table or guessing and testing, nor do they consist of additional contexts or meanings.
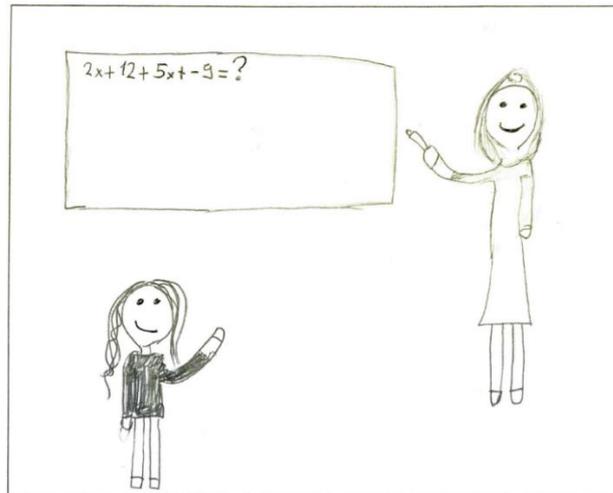
The teacher is asking the question on the board and I am responding. She is using board marker, I am using pencil and notebook. (S105, grade 7, girl)

Figure 5. S105's drawing and description of mathematics classroom
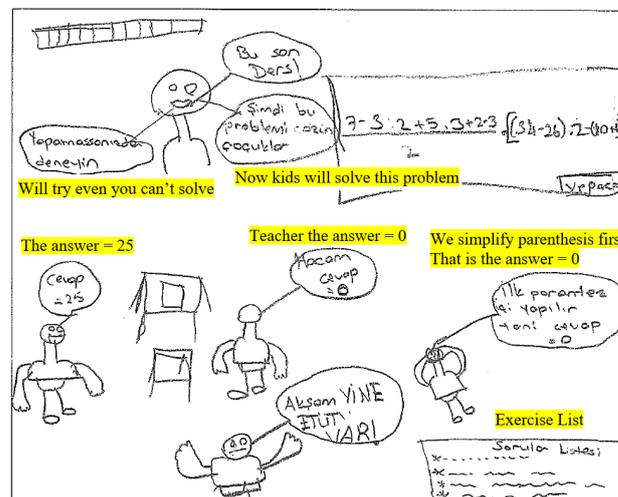
The teacher asked us a question, everybody is solving it. The teacher always helps us, and she is always equal to everyone, just a little much disciplined. I don't say these to all teachers, but only to a few. (S65, grade 7, boy)

Figure 6. S65's drawing and description of mathematics classroom

None of the students' visual or written responses involved a context in which a type of teaching was implemented based on representational, contextual, or open-ended tasks. Predominantly the drawings revealed closed mathematics exercises with one answer and depicted classroom experiences based on repetitious practice of similarly constructed examples. **Figure 4** includes the homework exercises that are being solved in the classroom, one of which is solving the equation $5x + 10 = 3x + 2$ for x. **Figures 5** and **6** present similar examples to closed mathematics exercises. **Figure 6** captures classroom experiences which require students to apply learned procedures to a set of similar problems. These figures provide a few representative examples, and exemplify the prevalence of such perceived classroom experiences in this research.

Mathematical Representations

Table 8 provides the frequency of mathematical work corresponding to the forms of representations depicted in student drawings. As can be seen, the form of representations in mathematical work depicted in the drawings is predominantly symbolic in the form of equations (n=70, 72.2%) or expressions (n=17, 17.5%). The mathematical work in three responses represented static pictures (visual): a number line (see **Table 8**); a cube (S59, grade 8, gender not provided); and a set diagram and graph (S81, grade 6, girl). In two drawings,

Table 8. The forms of mathematical representations used in student drawings

Representation form		Frequency	Example
Symbolic	Equations	70 (72.2%)	
	Numerical	49 (50.5%)	$2 \times -2 \div 4^2 \times 48^2 =?$ (S17, grade 8, girl)
	Algebraic	21 (21.7%)	$(\frac{3x}{b-1})^{-2} = x + b =?$ (S19, grade 8, boy)
	Expressions	17 (17.5%)	
	Numerical	9 (9.3%)	$2\sqrt{5}$; $4 + 18$ (S118, grade 8, boy)
	Algebraic	8 (8.2%)	$(2x + 5) \times (4x - 3)$ (S101, grade 7, boy)
Visual	Static pictures	3 (3.1%)	(S75, grade 6, girl)
Verbal	Specialised language related to: Probability; and	1 (1%)	Then 1 means certainty doesn't it? [the event will happen; its probability is 1] (S117, grade 8, girl)
	Square root	1 (1%)	A quadratic number is the square of a number. (S56, grade 8, boy)

verbal information is represented: in one response the probability of certain events is discussed, and in another the concept of square root is represented. In the remaining seven responses no reference was made to mathematical representations.

The students only depicted and described standard classroom materials such as the whiteboard, ruler, board markers, pencil, books, and in a few responses, a smartboard. Manipulatives, concrete materials or technological tools used in mathematics teaching and learning were absent in student responses. In relation to the use of mathematical tools, S107 wrote: "*The teacher is lecturing, the student is listening, they are not using materials*" (grade 8, gender not provided).

DISCUSSION

As part of a larger study investigating Turkish middle school students' perceived experiences of their mathematics classrooms through examining their drawings on the Draw a Mathematics Classroom Test, this article has presented data about the types of mathematical tasks and forms of representations in the students' depictions. The findings extend existing literature from around the world, on perceived teaching and learning practices in mathematics classrooms (e.g., Echazarra, Salinas, Méndez, Denis, & Rech, 2016; O'Dwyer, Wang, & Shields, 2015; OECD, 2016) by providing the analysis of data from student drawings of mathematics classrooms in Turkey.

While I acknowledge that participant student drawings might not reflect their actual classroom experiences, I believe, through the drawing method, they were able to draw freely about their experiences of mathematics teaching and learning (Kearney & Hyle, 2004). They were given control of the data collection process. I suggest that associated with this control was an increased likelihood that students would depict authentic pictures of their perspectives about teaching and learning of mathematics. Even the latest changes in the regulations relating to clothing and appearance of school employees (in this case teachers) in Turkey, could be observed in student drawings. That is, until relatively recently, female teachers were not allowed to wear headscarves in class and in my previous drawing research (e.g., Hatisaru, 2019a), no students drew their female teachers with scarves. In this study, which was implemented after the regulations changed, several students depicted female teachers wearing headscarves. I take this observation as evidence that students do indeed draw what they see and experience, and hence substantiates the effectiveness of the drawing method as a means of investigating student views about mathematics and its teaching and learning. Additionally, studies designed to validate whether students' depictions were representative of their actual classroom experiences, through incorporating teacher interviews and classroom observations (Remesal, 2009) or classroom video recordings (Laine et al., 2020), have found a close link between the two: student drawings and the actual classroom practices. Remesal (2009) reported that:

> The main common result of these cases is the identification of young primary pupils'
> capability of perceiving assessment practices as ruled by distinctive norms and
> conventions in the classroom among other classroom routines: 'someone is to ask and
> someone is to respond,' 'someone is to show the work and someone is to mark the work,'

‘grades are given and the parents are informed.’ This awareness develops even though the teachers themselves might not believe 8-year olds are capable of such insights. (p. 47)

The outcomes of this study should be read in light of this caveat.

The drawing tasks provided by the students elicited little to no indication indicating that students experience a variety of learning experiences based on varied mathematical tasks. Procedural tasks dominated student responses, while representational, contextual, and open-ended tasks were mostly absent. Consistent with the previous research on student thoughts about mathematics classroom experiences (e.g., Hatisaru, 2019b; Picker & Berry, 2000; OECD, 2016), most of the students' responses reflected their perceptions of classroom experiences which emphasise the repetition of problems and the application of facts and procedures to solve routine problems. There was little evidence of classroom environments where the teaching and learning of mathematics incorporates tasks of sufficient richness to impact development of mathematical proficiency in students. From the perspective of Smith and Stein's (1998) taxonomy, the tasks identified in student responses are viewed as low-level tasks that involve procedures without connections (e.g., $2^x = 3, 8^x = ?$), since the level of student thinking required by those tasks is the application of learned procedures applied to a set of similar problems. It is likely that in their classrooms, students are expected to use certain learned or memorised procedures that leave almost no ambiguity about what they need to do in order to solve the respective task.

The results imply that in the depicted classrooms, one of the teachers' main intentions is to develop students' procedural skills through, initially demonstrating specific procedures to them and then making them practise similarly constructed examples. The predominance of such mathematical experiences is worrisome, because the development of students' mathematical understanding is as important as the development of their procedural competences, and the former needs students to engage in mathematical tasks of sufficient richness and variety (National Governors Association Centre for Best Practices and Council of Chief State School Officers [NGA Centre & CCSSO], 2010). Instructional practices utilising rich mathematical tasks, presented in different contexts, that can be solved in different ways or for which there is no immediately obvious method have greater positive associations with conceptual understanding of mathematics in students, while procedural tasks contribute little to conceptual understanding (Blazar, 2015; Echazarra et al., 2016; OECD, 2016). Being confronted with mainly procedural tasks is also boring and restrictive for students (Sullivan, 2011). In time, students can come to see mathematics as mostly numbers or formulae (Boaler, 2015), as was evident in the students views in the research by Hatisaru (2019a) and Ucar et al. (2010), which contributes to their disengagement with mathematics (Goldin, 2018).

Teachers are responsible for the creation and maintenance of the mathematics learning environment experienced by students such as those who participated in this study. Teachers make choices about the rules, procedures and domain-specific methods in mathematical practices (Goldin, 2018) incorporated in the learning environment, based upon their particular beliefs about, and skills for teaching and learning of mathematics. In Turkey, and globally, some teachers view mathematics as a body of knowledge to practice and understand, and value procedural competency more (Yildirim & Yildirim, 2019). Compared to the more diverse pedagogies such as problem/project-based approach, inquiry or collaborative learning, the traditional direct teaching instruction is the most commonly used pedagogy in the teaching of mathematics (Nistor, Gras-Velazquez, Billon, & Mihai, 2018). Among other instructional practices, tasks for which there is no immediately obvious solution and require students' cognitive activation and form them to think critically are less commonly practiced by teachers in the classroom. Many students encounter complex, real-life based problems with multiple solution far less than tasks which require them to memorise and apply facts and procedures (Vincent-Lancrin, Urgel, Kar, & Jacotin, 2019).

On the other hand, "Choices about the teaching and learning of mathematics also depend on what society wants educated adults to know. Questions of what needs to be taught are essentially questions of what knowledge is most preferred." (Kilpatrick et al., 2001, p. 21). In Turkey (Altinyelken, 2011), and internationally (Nistor et al., 2018; Vincent-Lancrin et al., 2019), the prevalence of mathematics classes such as the ones documented in this study, can be related to the assessments that dominate the education system and the existence of high-stakes, national tests. In Turkey, being able to apply a formula and get the correct answer to questions is necessary and sufficient for success in both classroom assessments and in high-stakes tests. Such tests are highly competitive, and parents perceive students' performance in them to be very important (Altinyelken & Sozeri, 2017) as the results determine which high school or university a student will go to. To be placed in relatively good schools and universities, students work very hard; they practice hundreds of questions either at the school in mathematics classes, or during out-of-school instruction. Predominant

mathematical practices could be evidence of a focus on short-term gains (exam results) rather than educating learners of mathematics (Goldin, 2018; Malkevitch, 1997; Watson & De Geest, 2005). I suggest that the potential negative impact of high-stakes testing on the teaching and learning approaches and the mathematical tasks experienced in Turkish classrooms warrants further research. Specifically, the design of explanatory studies which investigate both how mathematical practices in the classroom are related to high-stakes testing, and how students' classroom experiences associate with their learning outcomes.

As anticipated, teaching and learning activities in effective mathematics classrooms are structured both to use different modes of representations (e.g., diagrams, tables, concrete models, real life contexts, written symbols) and to enable students to translate between these representational modes (Lesh et al., 2003; NCTM, 2010, 2014). In this study, however, the analysis showed that the most common way students perceived that mathematics is represented is through symbolic representations. In only five student responses were mathematical concepts relevant to numbers, geometry or probability presented through a visual or verbal representational mode. None of the drawings indicated students engaging in tasks or activities through which concepts or ideas relevant to mathematics could be presented through different representational modes. Also, the use of mathematical manipulatives and/or technology were not depicted in their classrooms.

A possible explanation for the lack of reference to the use of mathematical representations including physical tools within the drawings, can be found in the related literature. When comparing the Common Core State Standards for Mathematics [CCSSM] (2010) (NGO Centre & CCSSO, 2010) (e.g., using appropriate tools strategically, modelling with mathematics) in the learning areas of middle school mathematics curricula of Turkey and Singapore (e.g., Algebra, Probability), Erbilgin (2017) found that there is less emphasis on using multiple representations in Turkey than in Singapore. The Turkish curriculum suggests teachers use appropriate manipulatives and technology where applicable, leaving it as optional, as Erbilgin (2017) presumed, because those resources might not be available in all schools. Several studies from Turkey (Altinyelken, 2011; Altinyelken & Sozeri, 2017) have reported that a teacher's decision to incorporate particular curriculum aspects (in this case, the use of mathematical tools, including both representational and physical) into their classroom practice is influenced by several dimensions in education, including their beliefs, interpretations of the pedagogical approaches, and classroom realities (Altinyelken & Sozeri, 2017) as well as the prevalence of national high-stakes testing (Ozpolat, 2013). In the absence of observational or interview data, it is difficult to confirm that the curriculum guidance or availability of materials, or exams and the examination system, have influenced the student response patterns found in this study. More research on this topic needs to be undertaken before the associations between, for example teachers' belief about the use of multiple representations (Dreher, Kuntze, & Lerman, 2016) and mathematical practices can be understood.

IMPLICATIONS FOR PRACTICE AND RESEARCH

The results of this study provide implications for policy making, practice, and research. Perhaps the most important implication for research is that student drawings as a measure of perceptions of their experiences in mathematics classrooms have the potential to provide reliable information. As such, the research instrument in this study provides researchers with a tool to see, through the eyes of students, the mathematical practices used by teachers in mathematics classrooms. Having the participant students explain their pictures with clear and direct prompts (e.g., What are the students doing?) nicely complemented my own judgements. Therefore, I would encourage the use of descriptive methods (verbal explanations) together depictive methods (pictures) when perceptions are being researched as an alternative to interviews and/or classroom observations. In the analysis of drawings, identifying what is essential for the analysis in a clear and compelling way, and defining each component warrant for specificity in the analysis. Using deductive content analysis where the conceptualisation of data analysis is based on previous research has been informative.

Most of the students depicted their mathematics classroom in which similar algebraic problems are systematically practiced (e.g., Solve the equation $5x + 10 = 3x + 2$ for x), presumably for students getting high test scores. The implication of this finding is valuable to both practitioners and teacher educators. For practitioners, it seems imperative that school teachers (and parents) realise that achievement in mathematics does not necessarily mean high test scores. The focus of the learning experiences of school students should be moved beyond solving test questions correctly to meaningful learning; that is, being able to use mathematics meaningfully in their planning and decision-making (Goldin, 2018). Within the former focus, some perceptions that students can develop are that mathematics is formulae and computations, that to learn mathematics is to find a correct answer to questions quickly, and that the mathematical skills desired by industry for instance,

are computational (Ucar et al., 2010). It is far more difficult to envision students who have these perceptions and are not good at mathematical procedures will pursue mathematical problem solving with enthusiasm or embark on mathematics or mathematics-related careers (Goldin, 2018).

Teacher educators should be aware that working on rich mathematical tasks contributes to the development of mathematical proficiency in school students, and that working with representations (e.g., visual, symbolic) plays a critical role in helping them develop flexible thinking and problem solving, and provides multiple entry points and access to the study of mathematics (Huinker, 2015). The ability to provide effective mathematics teaching, inclusive of rich tasks and explicit strategies to develop students' representational competence, is one element of teacher professional knowledge. Expertise in teacher education graduates in these areas is key to achieving desirable mathematics learning outcomes in school students.

Findings reveal that policy makers should become concern about whether Turkish school students have been well prepared for the demands of the 21st century. The classroom practices depicted by students in this research have shown that despite the major research-informed curriculum reforms (MoNE, 2018), little change could be seen in the classroom in terms of teaching and learning practices (see the Context section). It is useful to policy makers to assess whether as high-stakes testing has come to be focus of the society, the issue of mathematical understanding has become salient or not, and to identify (besides curriculum revisions) what additional components are essential to improve mathematical practices in the classroom. Providing teachers quality trainings organised according to their needs is vital (Directorate of Teacher Education and Development, 2017). The data reported here points to the need for professional development designed for mathematics teachers, aimed at building their capacity to understanding of the nature of mathematical tasks and to the use of representations in the teaching and learning of mathematics.

ACKNOWLEDGEMENTS

Disclosure statement

No potential conflict of interest was reported by the author.

Notes on contributors

Vesife Hatisaru - University of Tasmania, Australia.

REFERENCES

Aguilar, M. S., Rosas, A., Zavaleta, J., & Romo-Vázquez, A. (2016). Exploring high-achieving students' images of mathematicians. *International Journal of Science and Mathematics Education, 14*(3), 527-548. https://doi.org/10.1007/s10763-014-9586-1

Altinyelken, H. K. (2011). Student-centred pedagogy in Turkey: Conceptualizations, interpretations and practices. *Journal of Education Policy, 26*(2), 137-160. https://doi.org/10.1080/02680939.2010.504886

Altinyelken, H. K., & Sozeri, S. (2017). Assumptions and implications of adopting educational ideas from the west: the case of student-centered pedagogy in Turkey. In M. Akiba & G. LeTendre (Eds.), *International Handbook of Teacher Quality and Policy* (pp. 254-270). Routledge: New York. https://doi.org/10.4324/9781315710068-17

Anthony, G., & Walshaw, M. (2009). *Effective pedagogy in mathematics*. Educational Series 19. Brussels: International Academy of Education; Geneva: International Bureau of Education.

Australian Curriculum, Assessment and Reporting Authority (ACARA). Australian Curriculum: Mathematics. Retrieved from https://www.australiancurriculum.edu.au/f-10-curriculum/mathematics/

Blazar, D. (2015). Effective teaching in elementary mathematics: Identifying classroom practices that support student achievement. *Economics of Education Review, 48*, 16-29. https://doi.org/10.1016/j.econedurev.2015.05.005

Boaler, Jo. (2015). *What's math got to do with it? How teachers and parents can transform mathematics learning and inspire success.* New York, New York: Penguin Books.

Bobis, J., Anderson, J., Martin, A., & Way, J. (2011). A model for mathematics instruction to enhance student motivation and engagement. In D. J. Brahier & W. R. Speer (Eds.), *Motivation and disposition: Pathways to learning, 73rd yearbook of the National Council of Teachers of Mathematics* (pp. 1-12). Reston, VA: NCTM.

Bragg, L. (2007). Students' conflicting attitudes towards games as a vehicle for learning mathematics: A methodological dilemma. *Mathematics Education Research Journal, 19*(1), 29-44. https://doi.org/10.1007/BF03217448

Brown, L. (1992). The influence of teachers on children's image of mathematics. *For the Learning of Mathematics, 12*(2), 29-33.

Chambers, D. W. (1983). Stereotypic images of the scientist: The Draw-a-Scientist Test. *Science Education, 67*(2), 255-265. https://doi.org/10.1002/sce.3730670213

Cramer, K. (2003). Using a translation model for curriculum development and classroom instruction. In R. Lesh & H. Doerr (Eds.), *Beyond constructivism. Models and modeling perspectives on mathematics problem solving, learning, and teaching.* Mahwah, NJ: Lawrence Erlbaum.

Dreher, A., Kuntze, S., & Lerman, S. (2016). Why use multiple representations in the mathematics classroom? Views of English and German preservice teachers. *International Journal of Science and Mathematics Education, 14*(2), 363-381. https://doi.org/10.1007/s10763-015-9633-6

Echazarra, A., Salinas, D., Méndez, I., Denis, V., & Rech, G. (2016). How teachers teach and students learn: Successful strategies for school. *OECD Education Working Papers, No. 130.* OECD Publishing, Paris. https://doi.org/10.1787/5jm29kpt0xxx-en

Elo, S., & Kyngäs (2008). The qualitative content analysis process. *Journal of Advanced Nursing, 62*(1), 107-115. https://doi.org/10.1111/j.1365-2648.2007.04569.x

Erbilgin, E. (2017). A comparison of the mathematical processes embedded in the content standards of Turkey and Singapore. *Research in Social Sciences and Technology, 2*(1), 53-74. https://doi.org/10.46303/ressat.02.01.3

Ernest, P. (2010). Add it up. Why teach mathematics? *Professional Educator, 9*(2), 44-47.

European Schoolnet (October, 2018). *Science, technology, engineering and mathematics education policies in Europe. Scientix observatory report.* European Schoolnet, Brussels.

Eurydice (2019a). *Initial Education for Teachers Working in Early Childhood and School Education.* Retrieved from https://eacea.ec.europa.eu/national-policies/eurydice/content/turkey_en

Eurydice (2019b). *Continuing Professional Development for Teachers Working in Early Childhood and School Education.* Retrieved from https://eacea.ec.europa.eu/national-policies/eurydice/content/turkey_en

Fraser, B. (2014). Classroom learning environments: Historical and contemporary perspectives. In Lederman, N. & Abell, S. (Eds), *Handbook of Research on Science Education Volume II* (pp. 104-119). USA: Routledge.

Finson, K. D., Beaver, J. B., & Cramond, B. L. (1995). Development and field test of a checklist for the Draw-a-Scientist Test. *School Science and Mathematics, 95*(4), 195-205. https://doi.org/10.1111/j.1949-8594.1995.tb15762.x

Goldin, G. A. (2004). Problem solving heuristics, affect, and discrete mathematics. *ZDM, 36*(2), 56-60. https://doi.org/10.1007/BF02655759

Goldin, G. A. (2014). Mathematical Representations. In Lerman S. (Eds), *Encyclopedia of Mathematics Education.* Springer, Dordrecht. https://doi.org/10.1007/978-94-007-4978-8_103

Goldin, G. A. (2018). Discrete mathematics and the affective dimension of mathematical learning and engagement. In E. W. Hart, & J. Sandefur (Eds), *Teaching and learning discrete mathematics worldwide: Curriculum and research* (pp. 53-65). Cham: Springer; 2017. p. 53-65 https://doi.org/10.1007/978-3-319-70308-4_4

Goodenough, F. L. (1926). *Measurements of intelligence by drawings.* World Book Co.

Gulek, C. (1999). *Using multiple means of inquiry to gain insight into classrooms: A multi-trait multi-method approach* (Unpublished doctoral dissertation). Boston College, Chestnut Hill, MA.

Haney, W., Russel, M., & Bebell, D. (2004). Drawing on education: Using drawings to document schooling and support change. *Harvard Educational Review, 74*(3), 241-271. https://doi.org/10.17763/haer.74.3.w0817u84w7452011

Harris, L. R., Harnett, J. A., & Brown, G. (2009). "Drawing" out student conceptions: Using pupils' pictures to examine their conceptions of assessment. In D. M. McInerney, G. T. L. Brown, & G. A. D. Liem (Eds.), *Students perspectives on assessment: What students can tell us about assessment for learning* (pp. 53-83). Charlotte, NC: Information age publishing, Inc.

Hatisaru, V. (2018). Teachers' beliefs about knowledge of teaching and their impact on teaching practices. In Rott, B., Torner, G., Peters-Dasdemir, J., Moller, A., & Safrudiannur (Eds), *Views and Beliefs in Mathematics Education: The Role of Beliefs in the Classroom* (pp. 149-159). Switzerland: Springer.

Hatisaru, V. (2019a). Lower secondary students' views about mathematicians depicted as mathematics teachers. *LUMAT: International Journal on Math, Science and Technology Education, 7*(2), 27-49. https://doi.org/10.31129/LUMAT.7.2.355

Hatisaru, V. (2019b). Putting the spotlight on mathematics classrooms. In J. Novotná & H. Moraová (Eds.), *Proceedings of the International Symposium Elementary Mathematics Teaching* (pp. 182–192). ISBN 9788076030695.

Hatisaru, V. (2020). Perceived need for mathematics among lower secondary students. *Australian Mathematics Education Journal, 2*(1), 9-14.

Hatisaru, V. (submitted). School Students' Depictions of Mathematics Teaching and Learning Practices. *International Journal of Science and Mathematics Education*.

Hatisaru, V., & Erbaş, A. K. (2017). Mathematical knowledge for teaching and student learning outcomes. *International Journal of Science and Mathematics Education, 15*(4), 703-722.

Hatisaru, V., & Murphy, C. (2019). 'Creature' teachers 'monster' mathematicians: students' views about mathematicians and their stated attitudes to mathematics. *International Journal of Education in Mathematics, Science and Technology, 7*(3), 215-221.

Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Greenwich, CT: Information Age.

Huinker, D. (2015). Representational competence: a renewed focus for classroom practice in mathematics. *Wisconsin Teacher of Mathematics, 67*(2), 4-8

Johansson, D. A., & Sumpter, L. (2010). Childrens' conceptions about mathematics and mathematics education. In K. Kislenko (Ed.), *Proceedings of the MAVI-16 conference June 26-29, 2010* (pp. 77-88). Tallinn, Estonia: Tallinn University of Applied Sciences.

Johnson, E. L. (2018). *A New Look at the Representations for Mathematical Concepts: Expanding on Lesh's Model of Representations of Mathematical Concepts*. Forum Public Policy Online 11.

Kearney, K. S., & Hyle, A. E. (2004). Drawing out emotions: The use of participant-produced drawings in qualitative inquiry. *Qualitative Research, 4*(3), 361-382. https://doi.org/10.1177/1468794104047234

Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Laine, A., Näveri, L., Ahtee, M., Hannula, M. S., & Pehkonen, E. (2013). Emotional atmosphere in third graders' mathematics classroom - An analysis of pupils' drawings. *Nordic Studies in Mathematics Education, 17*(3-4), 101-116.

Laine, A., Ahtee, M., & Näveri, L. (2020). Impact of teachers' actions on emotional atmosphere in mathematics lessons in primary school. *International Journal of Science and Mathematics Education, 18*, 163-181. https://doi.org/10.1007/s10763-018-09948-x

Lane, C., Stynes, M., & O'Donoghue, J. (2014). The image of mathematics held by Irish post-primary students. *International Journal of Mathematical Education in Science and Technology, 45*(6), 879-891. https://doi.org/10.1080/0020739X.2014.884648

Latterell, C. M., & Wilson, J. L. (2012). Students' perceptions of what mathematicians do. *The Mathematics Educator, 13*(2), 73-84.

Lesh R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of Representation in the Teaching and Learning of Mathematics* (pp. 33-40) Hillsdale, NJ: Lawrence Erlbaum Associates.

Lesh, R., Cramer, K., Doerr, H., Post, T., & Zawojewski, J., (2003) Using a translation model for curriculum development and classroom instruction. In R. Lesh & H. Doerr (Eds.) *Beyond Constructivism. Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching*. Lawrence Erlbaum Associates, Mahwah, New Jersey. https://doi.org/10.4324/9781410607713

Losh, S. C., Wilke, R., & Pop, M. (2008). Some methodological issues with "Draw a Scientist Tests" among young children. *International Journal of Science Education, 30*(6), 773-792. https://doi.org/10.1080/09500690701250452

Malkevitch, J. (1997). Discrete mathematics and public perceptions of mathematics. In J. G. Rosenstein, D. S. Franzblau, & F. S. Roberts (Eds.), *Discrete Mathematics in the Schools* (pp. 89-97). American Mathematical Society/NCTM, Providence, RI. https://doi.org/10.1090/dimacs/036/10

Mead, M., & Metraux, R. (1957). Image of the scientist among high school students: A pilot study. *Science, 126*, 384-390. https://doi.org/10.1126/science.126.3270.384

Ministry of National Education (2018). *Matematik dersi öğretim programı (İlkokul ve Ortaokul 1, 2, 3, 4, 5, 6, 7 ve 8. Sınıflar) [Mathematics curriculum (Primary and Lower Secondary School Grades 1, 2, 3, 4, 5, 6, 7, and 8)]*. Retrieved from: http://mufredat.meb.gov.tr/

Mullis, I. V. S., Martin, M. O., Foy, P., & Hooper, M. (2016). *TIMSS 2015 international results in mathematics*. Retrieved from Boston College, TIMSS & PIRLS International Study Center website: http://timssandpirls.bc.edu/timss2015/international-results/

Murphy, P. K., Delli, L. A. M., & Edwards, M. N. (2004). The good teacher and good teaching. Comparing the beliefs of second-grade students, preservice teachers, and inservice teachers. *The Journal of Experimental Education, 72*(2), 69-92. https://doi.org/10.3200/JEXE.72.2.69-92

Narayan, R., Park, S., Peker, D., & Suh, J. (2013). Students' images of scientists and doing science: an international comparison study. *Eurasia Journal of Mathematics, Science & Technology Education, 9*(2), 115-129. https://doi.org/10.12973/eurasia.2013.923a

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston VA: Author.

National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all*. Reston VA: Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Author. http://www.corestandards.org

Nesin, A. (2014, August). 2023 icin cok gec, belki 2073! [Too late for 2023, perhaps 2073!]. *Gorus, 85*, 24-26.

Nistor, A., Gras-Velazquez, A., Billon, N., & Mihai, G. (December 2018). Science, technology, engineering and mathematics education practices in Europe. *Scientix Observatory Report*. European Schoolnet, Brussels.

O'Dwyer, L., Wang, Y., & Shields, K. (2015). Teaching for conceptual understanding: A cross-national comparison of the relationship between teachers' instructional practices and student achievement in mathematics. *Large-scale Assessments in Education, 3*(1), 1-30. https://doi.org/10.1186/s40536-014-0011-6

Organisation for Economic Co-operation and Development (OECD) (2016). *Ten questions for mathematics teachers ... and how PISA can help answer them*. PISA, OECD Publishing, Paris.

Ozdemir, E., Gonen, E., Polat, M., & Ari, S. A. (2015). Turkey: Overview of education system. *TIMSS 2015 Encyclopedia: Education Policy and Curriculum in Mathematics and Science*. Retrieved from Boston College, TIMSS & PIRLS International Study Center website: http://timssandpirls.bc.edu/timss2015/encyclopedia/

Ozpolat, V. (2013). The place of student-centered approach in teachers' occupational priorities. *National Education Journal, 43*(200), 5-27.

Pape, S. J., & Tchoshanov, M. A. (2001). The Role of Representation(s) in Developing Mathematical Understanding. *Theory into practice, 40*(2), 118-127. https://doi.org/10.1207/s15430421tip4002_6

Pehkonen, E., Ahtee, M., & Laine, A. (2016). Pupils' drawings as a research tool in mathematical problem-solving lessons. In P. Felmer, E. Pehkonen, & J. Kilpatrick (Eds.), *Posing and solving mathematical problems: Advances and new perspectives (Research in mathematics education)* (pp. 167-188). Cham, Switzerland: Springer. https://doi.org/10.1007/978-3-319-28023-3_11

Picker, S., & Berry, J., (2000). Investigating pupils' images of mathematicians. *Educational Studies in Mathematics, 43*, 65-94. https://doi.org/10.1023/A:1017523230758

Picker, S., & Berry, J. (2001). Your students' images of mathematicians and mathematics. *Mathematics Teaching in the Middle School, 7*(4), 202-208.

Remesal, A. (2009). Accessing primary pupils' conceptions of daily classroom assessment practices. In D. M. McInerney, G. T. L. Brown, & G. A. D. Liem (Eds.), *Students perspectives on assessment: What students can tell us about assessment for learning* (pp. 25-51). Charlotte, NC: Information age publishing, Inc.

Rock, D., & Shaw, J. M. (2000). Exploring children's thinking about mathematicians and their work. *Teaching Children Mathematics, 6*(9), 550-555.

Sam, L. C., & Ernest, P. (2000). A survey of public images of mathematics. *Research in Mathematics Education, 2*(1), 193-206. https://doi.org/10.1080/14794800008520076

Saunders, B., Sim, J., Kingstone, T., Baker, S., Waterfield, J., Bartlam, B., … Jinks, C. (2017). Saturation in qualitative research: Exploring its conceptualization and operationalization. *Quality and Quantity, 52*, 1893-1907. https://doi.org/10.1007/s11135-017-0574-8

Smith, M. S., & Stein, M. K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School, 3*(5), 344-50.

Stein, M. K., & Smith, M. S. (2011). *Mathematical tasks as a framework for reflection: From research to practice*. National Council of Teachers of Mathematics (NCTM). Reston, Va.: NCTM.

Stiles, D. A., Adkisson, J. L., Sebben, D., & Tamashiro, R. (2008). Pictures of hearts and daggers: Emotions are expressed in young adolescents' drawings of their attitudes towards mathematics. *World Cultures eJournal, 16*(2).

Sullivan, P. (2011). *Teaching mathematics: Using research-informed strategies*. Camberwell, VIC: Australian Council for Educational Research.

Swan, M. (2005). *Improving learning in mathematics: Challenges and strategies*. Sheffield, England: Department of Education and Skills Standards Unity.

Thomas, J. A., Pedersen, J. E., & Finson, K. (2001). Validating the Draw-A-Science-Teacher-Test Checklist: Exploring mental models and teacher beliefs. *Journal of Science Teacher Education, 12*(4), 295-310. https://doi.org/10.1023/A:1014216328867

Tripathi, P. (2008). Developing mathematical understanding through multiple representations. *Mathematics Teaching in the Middle School, 13*(8), pp. 438-445.

Turkish Directorate of Teacher Education and Development (2017). *Teacher 2017-2023 Strategy Document*. Retrieved from http://oygm.meb.gov.tr/www/ogretmen-strateji-belgesi/icerik/406

Ucar, Z., Piskin, M., Akkas, E., & Tascı, D. (2010). Elementary students' beliefs about mathematics, mathematics teachers and mathematicians. *Education and Science, 35*(155), 131-144.

Vieluf S., Kaplan, D., Klieme, E., & Bayer, S. (2012). *Teaching practices and pedagogical innovation: Evidence from TALIS*. OECD Publishing. https://doi.org/10.1787/9789264123540-en

Watson, A., & De Geest, E. (2005). Principled teaching for deep progress: Improving mathematical learning beyond methods and materials. *Educational Studies in Mathematics, 58*(2), 209-234. https://doi.org/10.1007/s10649-005-2756-x

Wilson, P. (2011). Disposition towards engagement in mathematics. In Smith, C. (Ed), *Proceedings of the British Society for Research into Learning Mathematics, 31*(2), 67-72.

Vincent-Lancrin, S., Urgel, J., Kar, S., & Jacotin, G. (2019). Measuring innovation in education 2019: What has changed in the classroom? *Educational Research and Innovation*, OECD Publishing, Paris. https://doi.org/10.1787/9789264311671-en

Wong, N. Y., Marton, F., Wong, K. M., & Lam, C. C. (2002). The lived space of mathematics learning. *Journal of Mathematical Behavior, 21*, 25-47. https://doi.org/10.1016/S0732-3123(02)00101-3

Yıldırım, S., & Yıldırım, H. H. (2019) Conceptions of Turkish mathematics teachers about the effectiveness of classroom teaching. *International Journal of Mathematical Education in Science and Technology, 50*(8), 1152-1165. https://doi.org/10.1080/0020739X.2019.1579929

APPENDIX A

Draw a Mathematics Classroom Test

Dear Students;

This survey aims to explore your perceptions of teaching and learning practices in mathematics classrooms. Your school name or your or your teacher's name is not needed. Your drawing and responses are confidential and only used for this study. Thank you for your participation.

Think about teachers of mathematics and the kinds of things you do in mathematics classrooms. Draw a picture of your teacher teaching and yourself learning.

Look back at the drawing and explain your drawing so that anyone looking at it could understand what your drawing means. What is the teacher doing? What are the students doing? What materials and tools are they using?

http://www.iejme.com