Development of the Scaled Boundary Finite Element Method for Image-Based Slope Stability Analysis

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Abstract This paper presents a numerical technique for geotechnical slope stability analysis, integrating digital image meshing with the scaled boundary finite element method, allowing site conditions such as complex stratigraphies, surface and internal geometry evolution to be simulated in a robust and straightforward procedure. The quadtree decomposition technique is used to automatically discretise the geometry directly from digital images using pixel information to accurately capture boundaries with fine-scale elements. The process allows complex numerical models to be generated from cross-section images of slopes, capitalising on the combination of the scaled boundary finite element method and quadtree meshing. The spatial distribution of the soil material properties can be represented by the colour of each pixel. A mapping technique is developed to integrate these parameters into the computational mesh. The feasibility of the proposed method is presented through case study simulations of an active large Australian open-pit mine, considering various aspects of complex features such as geometry, stratigraphy and material behaviour.

Keywords Scaled boundary finite element method, Slope stability, Spatial variation, Image-based analysis, Quadtree mesh

1 **1. Introduction**

2 Slope stability analysis is a critical problem in the open-pit mining industry where large-scale slope failure can
3 result in catastrophic consequences to human life, the environment and the economy. Factors such as slope
4 geometry, soil and rock properties, groundwater, geological settings and discontinuities may impinge on the
5 behaviour of soil structures and initiate slope instability (Niroumand et al., 2012) . The consequences of slope
6 failure directly impact economic and safety aspects of open-pit mines such as in the 2007 Yallourn mine incident
7 (Sullivan, 2008), which recorded a coal slope failure displacing approximately 6 million tonnes of brown coal
8 and the 2013 Bingham Canyon mine incident in Utah (Pankow et al., 2014), USA, which recorded the most
9 extensive “man-made” slope failure ever, resulting in 65 million cubic meters of material sliding to the floor of
10 the pit.

11
12 Various slope stability analysis methods often categorised into Limit Equilibrium (LE) and numerical methods,
13 have been developed over recent decades (Bishop, 1955; Chang, 1992; Griffiths & Lane, 1999; Janbu, 1954;
14 Lorig & Varona, 2000). However, the modelling of slope stability with consideration of realistic practical
15 conditions, continues to be a challenging task due to the complexity of the geology, hydrology and geometry of
16 the slopes. The majority of slope stability analyses are made on the assumption of homogeneous slopes using
17 deterministic methods (Griffiths & Lane, 1999; Reale et al., 2015) which does not reflect the reality of geological
18 uncertainties, complex conditions in open pit excavations (Deliveris et al., 2016) and the spatial variation of soil
19 material properties (Lacasse & Nadim, 1996). The variation of complex model properties such as fluctuating
20 stratigraphy and spatial material variation can have a considerable effect on slope stability and the factor of safety
21 (FoS) (Allan et al., 2012; Griffiths et al., 2009).

22
23 Some of the earliest techniques used to perform slope stability analysis were based on LE methods (Bishop, 1955;
24 Janbu, 1954). LE methods assume elastic, isotropic materials, time-independent behaviour and quasi-static
25 loading and cannot exhibit fully coupled hydro-mechanical behaviour to model stress-dependent effects of

groundwater (Bobet, 2010). However, the LE methods have been developed over time to consider spatially variable materials (Izadi et al., 2020) and groundwater distributions in the slope stability analysis (Lam & Fredlund, 1993).

Since rocks and soils in reality exhibit complex behaviour under different loads, numerical methods are often required to generate accurate results. The finite element method is the most common numerical method adopted to assess slope stability due to its versatility (Griffiths & Lane, 1999). Application of the finite element method requires generation of a computational mesh. However, incorporating time dependent model changes to a finite element mesh is a challenging task when the stratigraphy contains numerous material layers with intersecting boundaries (Berre et al., 2019; Ghadrddan et al., 2021; Vu et al., 2014). This necessitates significantly increased preprocessing time and human intervention.

Soil and rock layers are often assumed to be homogeneous in most slope stability analyses. However, more realistic simulations of slope stability necessitate spatially variable soil parameters in order to capture the inherent variations of soil and ground conditions. LEM has been used in combination with random field theory (Babu & Mukesh, 2004; Cho, 2007), however, the method is incapable of producing the stress and strains within the domain. The Random Finite Element Method (RFEM) has been widely applied to study characteristics of spatially variable material properties within slopes, where material parameters are determined by a random distribution function and mapped on a mesh for FEM simulation (Dyson & Tolooiyan, 2019a; Griffiths & Fenton, 2004; Griffiths et al., 2009; Le et al., 2015). However, this is often a complicated process as the RFEM often requires complex mesh generation algorithms to isolate and identify elements in the mesh with varying spatial material properties (Dyson & Tolooiyan, 2019a, 2020).

The scaled boundary finite element method (SBFEM) is a semi-analytical technique that was introduced in the late 1990's to model problems of unbounded media (Wolf & Song, 2000). Since its inception, the SBFEM has

1 been further developed for applications in many different fields in engineering including fracture propagation (He
2 et al., 2021), heat conduction problems (Yu et al., 2021), electrostatic problems (Liu & Lin, 2012), acoustic
3 analysis (Liu et al., 2019), dispersion analysis (Gravenkamp et al., 2012) and seepage flow (Johari & Heydari,
4 2018). A salient feature of SBFEM is that it requires only a simple scaling condition during the discretisation of
5 a domain into a computational mesh and hence enables its formulation on any arbitrary polygon (Ooi et al., 2018)
6 or quadtree of scales varying from small to large (K. Chen et al., 2018; X. Chen et al., 2018). The latter feature
7 facilitates a reduction in the computational resources, the time required for analysis and the burden of mesh
8 generation.

9 To overcome the limitations of the finite element method in slope stability analysis associated with complex
10 geometry, stratigraphy and material behaviour, this paper presents the development of an image-based mesh
11 generation method for slope stability analysis, integrating SBFEM and quadtree decomposition. The salient
12 features of this technique include:

- 13
14 • Automatic meshing of slope geometries directly from digital images, considering the soil stratigraphy. Mesh
15 refinement can be performed in regions of interest in a straightforward manner.
- 16 • Efficient mapping of heterogeneous soil material properties onto a computational mesh using a pixel-based
17 approach. This addresses the technical difficulties associated with the finite element method which often
18 requires material parameters to be mapped to a uniform mesh, resulting in significant and unnecessary
19 computational costs.
- 20 • Changes to model geometries such as slope excavation, backfilling and stratigraphic updates can be
21 conveniently addressed through simple changes in the digital image input. Traditional finite element method
22 requires surface and stratigraphic characteristics to mapped or drawn within the model engine. The procedure
23 is often time consuming and requires significant human interaction and pre-processing.

The research presented in this paper discusses the image based quadtree mesh generation method and integration of spatial variation strength parameters, formulation of SBFEM in the context of material nonlinearity and a case study of the Yallourn mine considering spatial variation, excavation and backfilling scenarios.

2. Image-based quadtree mesh generation and random strata distribution

Modern computer graphics often utilise quadtree structures to visualise 2D bodies by recursively subdividing spatial data into 4 children cells (Friskén & Perry, 2002). Any computational procedure may (if possible) adopt a quadtree decomposition directly to perform calculations. However, a quadtree decomposition will often generate hanging nodes between adjacent cells in the mesh. As a result, standard numerical techniques such as the finite element method cannot be directly applied to quadtree meshes. The SBFEM, on the other hand, can be formulated directly on polygons with an arbitrary number of sides and can be directly applied to quadtree meshes (X. Chen et al., 2018). Digital images contain a vast range of geometrical information in the colour intensity of the pixels, which can be used to represent features such as material types and property distributions. Using this information, the image-based quadtree mesh generation can be fully automated, and significantly minimises the requirement of human intervention during mesh generation. Furthermore, the SBFEM is capable of automating the process of meshing models including the spatial variation of material properties, resulting in decreased computational requirements (Bazyar & Talebi, 2014).

2.1. Image-based quadtree mesh generation

Digital images are made up of pixels which can be used to store information about material types and distributions with geometrical shapes. The numeric values of each pixel in the image are stored as an image colour matrix, which is completed with a background colour to create a square matrix of $2^n \times 2^n$ (Saputra et al., 2017). As shown in Fig. 1, the recursive process of the quadtree image decomposition is used to split the digital image into uniformly coloured squares, thus creating a quadtree mesh that overlaps the entire image. The colour of each pixel

1 is used to define the material parameters. For example, Fig. 1a displays red, yellow and green colours that
 2 represent different materials.

3
 4 The maximum and the minimum size of elements in the final quadtree mesh can be selected accordingly. When
 5 the model contains thin geological layers, the finer details can be captured by reducing the minimum size of the
 6 element during quadtree decomposition. Due to its hierarchical data structure, the quadtree is particularly suited
 7 to bridge the transition between regions of finer to coarser element sizes in the mesh. In contrast, when compared
 8 with the finite element method using only elements in the standard library, the presence of thin geological layers
 9 would result in a very fine mesh or elements with a distorted shape that can affect the accuracy of the
 10 computations. The quadtree decomposition can easily be implemented for square matrices using the MATLAB
 11 R2020 inbuilt function 'qtdecomp'. A gradation of mesh sizes can be utilised to construct an appropriate mesh
 12 using the quadtree technique.

19 **Fig. 1.** Quadtree decomposition of the image matrix

20
 21 *2.2. Spatial variation strength parameters integration into a computational mesh*

22 In practice, soil and rock parameters are often inhomogeneous as one moves through the layers of the strata. In
 23 the case of slope stability analysis, neglecting spatially variable properties can lead to an over-estimation of the
 24 slope factor of safety and an underestimation of the probability of failure (Griffiths & Fenton, 2004). Numerical
 25 models that allow the incorporation of heterogeneous parameters are often required, permitting the soil material

parameters to have spatial variations. As a result, different regions of the mesh have to be assigned different material properties.

To consider the heterogeneity of the material parameters within soils in slope stability analysis, a mapping procedure is developed to integrate the spatial variation of material properties into the computational mesh. For the numerical studies in this paper, the soil cohesion and friction angle are assumed to vary over the domain of the slope where a dry and non-permeable condition is considered. Using this scenario as an example, there are three input digital images during mesh generation i.e. a digital image informing the stratigraphy of the soil, a digital image informing the spatial variation of the soil cohesion and a digital image informing the spatial variation of the soil friction angle. The mesh for the slope stability analysis was obtained by concurrently considering all three images during the quadtree decomposition. The mesh generation process is shown in Fig. 2. The size of the cells in a quadtree decomposition is controlled by specifying a threshold value for each of the three aforementioned images (Song, 2018). In general, due to its more homogeneous layout, the threshold value for the stratigraphy image is set to be larger than the threshold values for the soil cohesion and friction angle. This also prevents the resulting computational mesh that is generated from becoming too dense. The quadtree decomposition is then carried out for the three images simultaneously subject to condition of the user-specified threshold values.

Mesh generation from the images requires the minimum and maximum pixel sizes as inputs. The dimension of a pixel is relative with respect to the physical dimensions of the geometry considered and can be determined by linear scaling. The mesh density is thus controlled by minimum and maximum pixel sizes irrespective of the resolution of the image. In this study, the resolution of the images provided are approximately 512×128 pixels for the stratigraphy, and spatial material variation. The minimum and maximum pixel sizes adopted are 2^0 and 2^3 , respectively. Because the minimum cell size is the size of the pixel, the mapping process is one-to-one as shown in Fig. 2. The stratigraphic image is used to identify the material type whilst the spatially varying material

parameters are extracted from their respective images. For the cells in the computational mesh that are larger than the pixel size, a weighted average value of the material parameters is obtained before assigning it to cell as shown in Fig. 2.

Fig. 2. Computational mesh generation combining soil material parameters and spatially variable friction angle and cohesion parameters

3. SBFEM formulation for an arbitrary sided polygon

3.1. Polygon boundary discretisation and coordinate transformation

The SBFEM can be formulated using polygons with an arbitrary number of sides, subject to a direct visibility criterion which requires the entire boundary of the polygon to be directly visible from a point within the domain called the scaling centre (Song, 2018). Fig. 3 shows a generic representation of the SBFEM on a 6-node polygon and 5-node (quadtree) cell. For convex polygons like those considered in this study, we select the geometric centre as the scaling centre.

Fig. 3. Polygon or quadtree-element modeled by SBFEM

The polygon boundary is discretised with line elements shown in Fig. 4 with local coordinates of η ($-1 \leq \eta \leq 1$) along with the line element and ζ ($0 \leq \zeta \leq 1$) as radial coordinate shown in Fig. 3.

Fig. 4. Two nodes line element

The Cartesian coordinate of any point in the triangular section created between the scaling centre and the line element can be expressed in scaled boundary finite element coordinates as

$$x(\xi, \eta) = x_0 + \xi \mathbf{N}(\eta) \mathbf{x}_b \quad (1)$$

$$y(\xi, \eta) = y_0 + \xi \mathbf{N}(\eta) \mathbf{y}_b \quad (2)$$

where the nodal coordinates of the polygon are $\mathbf{x}_b = [x_1 \ x_2 \ x_3 \ \cdots \ x_n]^T$ and $\mathbf{y}_b = [y_1 \ y_2 \ y_3 \ \cdots \ y_n]^T$

The shape functions of line elements of the polygon boundary are defined by

$$\mathbf{N}(\eta) = [N_1(\eta), N_2(\eta), N_3(\eta) \cdots N_n(\eta)] \quad (3)$$

The displacement field of any point in the polygon is defined according to SBFEM coordinates by

$$\mathbf{u}(\xi, \eta) = \mathbb{N}_u(\eta) \mathbf{u}(\xi) \quad (4)$$

with

$$\mathbb{N}_u(\eta) = \begin{bmatrix} N_1(\eta) & 0 & N_2(\eta) & 0 & \cdots & N_n(\eta) & 0 \\ 0 & N_1(\eta) & 0 & N_2(\eta) & \cdots & 0 & N_n(\eta) \end{bmatrix} \quad (5)$$

The strain field of any point in the polygon is defined using the standard linear operator matrix and applying the scaled boundary finite element transformation (Wolf, 2003) as

$$\boldsymbol{\epsilon}(\xi, \eta) = \mathbb{B}_1(\eta) \mathbf{u}(\xi)_{,\xi} + \frac{1}{\xi} \mathbb{B}_2(\eta) \mathbf{u}(\xi) \quad (6)$$

where, $\mathbb{B}_1(\eta) = \mathbf{b}_1(\eta) \mathbb{N}_u(\eta)$ and $\mathbb{B}_2(\eta) = \mathbf{b}_2(\eta) \mathbb{N}_u(\eta)_{,\eta}$ are the strain-displacement matrices (Song, 2018)

with

$$\mathbf{b}_1(\eta) = \frac{1}{|\mathbf{J}(\eta)|} \begin{bmatrix} y_{\eta}(\eta)_{,\eta} & 0 \\ 0 & -x_{\eta}(\eta)_{,\eta} \\ -x_{\eta}(\eta)_{,\eta} & y_{\eta}(\eta)_{,\eta} \end{bmatrix} \quad (7)$$

$$\mathbf{b}_2(\eta) = \frac{1}{|\mathbf{J}(\eta)|} \begin{bmatrix} -y_{\eta}(\eta) & 0 \\ 0 & x_{\eta}(\eta) \\ x_{\eta}(\eta) & -y_{\eta}(\eta) \end{bmatrix} \quad (8)$$

where $\mathbf{J}(\eta)$ is the determinant of the Jacobian matrix required for coordinate transformation

$$\mathbf{J}(\eta) = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \eta} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \xi} \end{bmatrix} = \begin{bmatrix} x_{\eta}(\eta) & y_{\eta}(\eta) \\ x_{\eta}(\eta)_{,\eta} & y_{\eta}(\eta)_{,\eta} \end{bmatrix} \quad (9)$$

The equilibrium condition of each polygon can be derived from virtual work principle (Deeks & Wolf, 2002) resulting in

$$\mathbb{E}_0 \xi^2 \mathbf{u}(\xi)_{,\xi\xi} + (\mathbb{E}_0 - \mathbb{E}_1 + \mathbb{E}_1^T) \xi \mathbf{u}(\xi)_{,\xi} - \mathbb{E}_2 \mathbf{u}(\xi) = 0 \quad (10)$$

where $\mathbb{E}_0, \mathbb{E}_1$ and $\mathbb{E}_2$ are the coefficient matrices for the line elements on the boundary of the polygon that are related to material properties and line elements of polygon from which, their exact forms can be referred to in Song (2018).

For bounded domains such as those considered in this study, the solution of $\mathbf{u}(\xi)$ has the form (Song, 2018)

$$\mathbf{u}(\xi) = \mathbf{V}_u \xi^{-\mathbf{S}_n} \mathbf{c}_n \quad (11)$$

where $\mathbf{S}_n$ is a Schur matrix containing negative eigenvalues along its diagonal and $\mathbf{V}_u$ is a corresponding transformation matrix. These are obtained by performing a Schur decomposition of the Hamiltonian matrix $\mathbb{Z}$ defined as

$$\mathbb{Z} = \begin{bmatrix} \mathbb{E}_0^{-1} \mathbb{E}_1^T & -\mathbb{E}_0^{-1} \\ \mathbb{E}_1 \mathbb{E}_0^{-1} \mathbb{E}_1^T - \mathbb{E}_2 & -\mathbb{E}_1 \mathbb{E}_0^{-1} \end{bmatrix} \quad (12)$$

such that

$$\mathbb{Z} \begin{bmatrix} \mathbf{V}_u & \bar{\mathbf{V}}_u \\ \mathbf{V}_q & \bar{\mathbf{V}}_q \end{bmatrix} = \begin{bmatrix} \mathbf{V}_u & \bar{\mathbf{V}}_u \\ \mathbf{V}_q & \bar{\mathbf{V}}_q \end{bmatrix} \begin{bmatrix} \mathbf{S}_n & \\ & \mathbf{S}_p \end{bmatrix} \quad (13)$$

where $\mathbf{V}_u, \mathbf{V}_q, \bar{\mathbf{V}}_u$ and $\bar{\mathbf{V}}_q$ are transformation matrices and $\mathbf{S}_n$ and $\mathbf{S}_p$ are block-diagonal Schur matrices (Song, 2018).

The vector $\mathbf{c}_n$ Eq. (14) are integration constants that are determined from the nodal displacements on the boundary of the polygon $\mathbf{u}_b = \mathbf{u}(\xi = 1)$ as (Wolf, 2003)

$$\mathbf{c}_n = \mathbf{V}_u^{-1} \mathbf{u}_b \quad (14)$$

3.2. SBFEM shape function and strain field in terms of SBFEM shape function

The radial displacement function derived in the previous section is utilised to determine the shape function and the stress-strain relationship in terms of SBFEM. Substituting the Eq. (11) and Eq. (14) into Eq. (4) scaled boundary shape function can be extracted as (Ooi et al., 2014)

$$\Psi(\xi, \eta) = \mathbb{N}_u(\eta) \mathbf{V}_u \xi^{-\mathbf{S}_n} \mathbf{V}_u^{-1} \quad (15)$$

Hence, Eq. (4) can be rewritten as

$$\mathbf{u}(\xi, \eta) = \mathbf{\Psi}(\xi, \eta) \mathbf{u}_b \quad (16)$$

The strain-displacement relationship in terms of SBFEM can be developed by substituting Eq. (16) into the strain field Eq. (6) resulting in (Ooi et al., 2014)

$$\boldsymbol{\epsilon}(\xi, \eta) = \mathbb{B}(\xi, \eta) \mathbf{u}_b \quad (17)$$

where

$$\mathbb{B}(\xi, \eta) = \mathbf{V}_\epsilon(\eta) \xi^{-\mathbf{S}_n - \mathbf{I}} \mathbf{V}_u^{-1} \quad (18)$$

is simplified strain-displacement matrix.

where $\mathbf{V}_\epsilon(\eta)$ is strain mode with

$$\mathbf{V}_\epsilon(\eta) = (\mathbb{B}_1(\eta) \mathbf{V}_u [-\mathbf{S}_n] + \mathbb{B}_2(\eta) \mathbf{V}_u) \quad (19)$$

3.3. Nonlinear SBFEM elasto-plastic formulation

Eq. (4) and Eq. (16) shows that the displacement and strain fields can be expressed in terms of their nodal displacements while treating each polygon in a mesh as an “element” in the finite element sense. Therefore, standard finite element procedures can be adopted to treat the nonlinear elasto-plastic properties of the geomaterial. Following the virtual work theory, the nonlinear equilibrium conditions applicable to a polygon or quadtree cell can be expressed as (Owen & Hinton, 1980)

$$\int_V \delta \boldsymbol{\epsilon}^T(\xi, \eta) \Delta \boldsymbol{\sigma}(\xi, \eta) dV = \int_\Gamma \delta \mathbf{u}^T(\xi, \eta) \mathbf{f}_t d\Gamma + \int_V \delta \mathbf{u}^T(\xi, \eta) \mathbf{f}_b dV - \int_V \delta \boldsymbol{\epsilon}^T(\xi, \eta) \boldsymbol{\sigma}(\xi, \eta) dV \quad (20)$$

where $\delta \boldsymbol{\epsilon}(\xi, \eta)$ is virtual strain field, $\Delta \boldsymbol{\sigma}(\xi, \eta)$ is incremental stress field, $\delta \mathbf{u}(\xi, \eta)$ is virtual displacement field of nodal, $\mathbf{f}_t$ is the surface traction and $\mathbf{f}_b$ is the body force intensity.

Substituting Eq. (16), Eq. (17) and incremental stress strain relationship for elasto-plastic material $\Delta \boldsymbol{\sigma} = \mathbf{D}_{ep} \Delta \boldsymbol{\epsilon}$, where $\mathbf{D}_{ep}$ is the elasto-plastic constitutive matrix, into Eq. (20), principle of virtual work is expanded as

$$\left(\int_V \mathbb{B}^T(\xi, \eta) \mathbf{D}_{ep} \mathbb{B}(\xi, \eta) dV \right) \Delta \mathbf{u}_b = \int_\Gamma \mathbf{\Psi}^T(\xi, \eta) \mathbf{f}_t d\Gamma + \int_V \mathbf{\Psi}^T(\xi, \eta) \mathbf{f}_b dV - \int_V \mathbb{B}^T(\xi, \eta) \boldsymbol{\sigma}(\xi, \eta) dV \quad (21)$$

The Eq. (21) is simplified considering the terms as Eq. (22), which is a system of nonlinear equation for a polygon

$$\mathbf{K}_{ep} \Delta \mathbf{u}_b = \mathbf{R}_{ext} - \mathbf{R}_{int} \quad (22)$$

where

$$\mathbf{K}_{ep} = \int_V \mathbb{B}^T(\xi, \eta) \mathbf{D}_{ep} \mathbb{B}(\xi, \eta) dV \quad (23)$$

is elasto-plastic stiffness matrix of the polygon

$$\mathbf{R}_{ext} = \int_{\Gamma} \boldsymbol{\Psi}^T(\xi, \eta) \mathbf{f}_t d\Gamma + \int_V \boldsymbol{\Psi}^T(\xi, \eta) \mathbf{f}_b dV \quad (24)$$

is external load vector of the polygon

$$\mathbf{R}_{int} = \int_V \mathbb{B}^T(\xi, \eta) \boldsymbol{\sigma}(\xi, \eta) dV \quad (25)$$

is the internal load vector of the polygon. The evaluation of the stiffness matrix and load vectors follow the procedure outlined in (Ooi et al., 2014)

Eq. (22) is assembled polygon by polygon in a manner similar to the finite element method resulting in the global system of equations

$$\left(\sum_{i=1}^{npol} \mathbf{K}_{ep} \right) \Delta \mathbf{u}_b = \sum_{i=1}^{npol} (\mathbf{R}_{ext} - \mathbf{R}_{int}) \quad (26)$$

Elasto-plastic materials show nonlinear behaviour beyond the yield point and requires that the nonlinearity to be tracked through a return mapping algorithm (De Souza Neto et al., 2008). The modified Newton-Rapson method is adopted to solve the Eq. (26) and project the closest point for every increment in the elasto-plastic model until convergence is achieved. The procedure to treat the elasto-plastic behaviour follows from standard finite element approaches (De Souza Neto et al., 2008).

In this study, material of the slope is assumed to follow the Mohr-Coulomb failure criterion

$$F = (\sqrt{3} \cos \theta_0 - \sin \theta_0 \sin \phi) q - 3p \sin \phi - 3c \cos \phi = 0 \quad (27)$$

where F is the yield function, p , q and θ_0 are the stress invariants and, ϕ and c are internal friction angle and cohesion respectively.

The computation of constitutive tangent matrix for elasto-plastic material is then computed as

$$\mathbf{D}_{ep} = \mathbf{D} - \mathbf{D} \left(\frac{\partial F}{\partial \boldsymbol{\sigma}} \right) \left(\frac{\partial F}{\partial \boldsymbol{\sigma}} \right)^T \mathbf{D} \left(\mathbf{A} + \left(\frac{\partial F}{\partial \boldsymbol{\sigma}} \right)^T \mathbf{D} \left(\frac{\partial F}{\partial \boldsymbol{\sigma}} \right) \right)^{-1} \quad (28)$$

where, $\mathbf{D}$ is the elastic constitutive matrix, $\mathbf{A} = -\frac{1}{\Delta\lambda} \frac{\partial F}{\partial \kappa} d\kappa$, κ is the hardening parameter and $\Delta\lambda$ is the plastic multiplier. The elasto-plastic constitutive matrix $\mathbf{D}_{ep}$ is not constant within the polygon when the material is yielding. To calculate the elasto-plastic matrix within the framework of the scaled boundary finite element method, the technique proposed by Ooi et al. (2014) is employed. $\mathbf{D}_{ep}$ is calculated at the sampling (integration points) within a cell and then a least squares fit is performed so that the elasto-plastic matrix is expressed in the form of a matrix power function in scaled boundary finite element coordinates. This allows the elasto-plastic stiffness matrix $\mathbf{K}_{ep}$ to be integrated analytically.

3.4. Shear strength reduction technique

In the assessment of slope stability, it is common to quantify the stability through the calculation of the Factor of Safety (FoS). In current engineering practice, the shear strength reduction approach is one of the most common methods employed to calculate FoS (Dawson et al., 1999; Griffiths & Lane, 1999; Li et al., 2012). The concept of the shear strength reduction technique involves the iterative reduction of material strength parameters, namely the cohesion c_0 and friction angle ϕ_0 according to the strength reduction factor (SRF) until the slope fails. The strength parameters are sequentially reduced through the relations

$$c = \frac{c_0}{\text{SRF}} \quad (29)$$

$$\phi = \tan^{-1} \frac{\tan \phi_0}{\text{SRF}} \quad (30)$$

$$\text{FoS} = \frac{c_0}{c} = \frac{\tan \phi_0}{\tan \phi} \quad (31)$$

where c_0 and ϕ_0 are initial cohesion and initial friction angle respectively and c and ϕ are cohesion and friction angle used in each reduction step to identify the slope's FoS. As the minimum FoS value of a stable slope is 1.0, this value was chosen as the initial SRF value in the strength reduction process. An elasto-plastic analysis is then

carried out to simulate the deformation of the slope under self weight. In the event where the analysis converges, the SRF is increased and simulation is repeated with the new material properties calculated using Eq. (29) and (30). This process is repeated until slope failure is identified through a large increase in the deformation cause a non-convergence for a given number of iterations (Ghadrdan et al., 2021; Griffiths & Lane, 1999). A convergence tolerance and iteration ceiling limit of 0.0001 and 500, respectively, are considered for the elasto-plastic analysis process (Smith & Griffiths, 2004). In numerical analysis of slope stability, the FoS is usually reported up to 2 decimal places depending on the type of numerical technique employed (Zolkepli et al., 2019). However, for mining applications such as that considered in this study, FoS values reported up to 1 decimal place is deemed sufficient. Hence, in this study, an accuracy of one decimal place is considered and the SRF is increased progressively by 0.1 (Zhang et al., 2019) after every successful convergence of elasto-plastic model.

(a) Image-based slope stability analysis flow chart

(b) Structure chart and example code for Strength reduction code

Fig. 5. SBFEM slope stability analysis code flow chart

The flow chart and pseudocode in Fig. 5 describe the major steps involved in the calculation of the slope's FoS starting from slope mesh generation . A summary of the procedure is outlined as follows:

- A quadtree decomposition is applied to the images of the slope stratigraphy, spatial variation of cohesion and friction angle to generate the mesh.
- Material properties are assigned to each cell in the mesh according to colours.
- The strength reduction process is performed, as summarised in Fig. 5(b). The SBFEM elasto-plastic model simulates the behaviour of the slope, performing strength reduction until the failure state (identified by non-convergence for a given number of iterations) is achieved and the FoS is calculated.

4. Case study

1 The Yallourn open-pit brown coal mine is the second-largest open-pit mine in Australia and is located in the
2 Latrobe Valley, in the state of Victoria (Fig. 6). Brown coal was deposited in the Gippsland Basin during the
3 Eocene to Late Miocene periods as three stratigraphic layers, namely the Traralgon, Morwell and Yallourn
4 formations (Barton et al., 1993). The thickness of the Yallourn coal layer ranges from 40 to 88 m, underlying the
5 10 – 48 m thick overburden. A non-coal layer of 80 to 100 m thickness lies beneath the Yallourn layer, formed
6 mainly from clay, silt and thin coal splits. The Morwell coal layer extends deeper to a thickness of 100 m (Soliman
7 et al., 2007). The Gippsland basin consists of an estimated 65 billion tonnes of brown coal, representing 80% of
8 Victoria's brown coal resources (Barton et al., 1993). In recent years, a range of studies have focused on Yallourn
9 Mine slope stability analysis (Dyson & Tolooiyan, 2019b, 2020; Ghadrddan et al., 2021) and geological
10 investigations (Tolooiyan et al., 2019; Tolooiyan et al., 2020).

17 **Fig. 6.** Yallourn mine location and study area (a) Location of Yallourn mine in Australia (b) Studied
18 cross section of the Yallourn mine

19
20 The North-East batter cross-section of the Yallourn mine (Fig. 6b) is the focus of this research. The elastic
21 modulus (E), Poisson's ratio (ν), dry weight (γ), cohesion(c_0) and friction angle(ϕ), for both coal and non-coal
22 are listed in Table 1. These parameters were obtained from a series of laboratory tests, as detailed by Karami and
23 Tolooiyan (2020).

24

1 **Table 1.** Geotechnical parameters for Yallourn coal and non-coal materials

Material	Colour of Material	γ (kN/m ³)	E (MPa)	ν	Cohesion (kPa)	Friction Angle ϕ (°)
Coal	Red	11.4	40	0.2	150.72	27.28
Non-coal	Blue	20.15	52	0.3	31.81	23.67

2

3 To demonstrate the slope stability analysis simulation process with the image-based SBFEM method, first, the

4 deterministic case of Yallourn mine slope is analysed. Thereafter, the slope is analysed considering the spatial

5 variation of strength parameters using random fields of the cohesion and friction angle shear strength parameters.

6 The randomness of the spatial variation data were generated using the method described by Dyson and Tolooiyan

7 (2019a). Finally, the ability to appropriately analyse alternations to slope geometries through excavation and

8 backfilling scenarios is demonstrated.

9

10 *4.1. Deterministic case of Yallourn mine slope cross section*

11 Slope stability problems are considered as unconfined, and hence, the dilatancy angle is not considered a critical

12 factor (Griffiths & Lane, 1999). The corresponding value of dilatancy angle is therefore considered as zero

13 according to non-associated plastic flow, and it is assumed that no volumetric changes occur during the yielding

14 of materials. In the first example, the ability of SBFEM to assess deterministic cases is demonstrated. The

15 stratigraphy image of the cross-section of the slope, as shown in Fig. 7, is used to generate the quadtree mesh.

16 The mesh is generated by specifying a minimum and maximum cell sizes of 2^0 : 2^3 resulting in the mesh shown in

17 Fig. 8.

22 **Fig. 7-** Model cross-section dimensions of the Yallourn mine

Fig. 8- Image-based quadtree mesh with specific material properties

The mesh contains 6811 quadtree elements and 8332 nodes. The quadtree mesh is generated considering pixel values corresponding to red (indicating coal) and blue (indicating the non-coal interseam). The material properties of coal and non-coal, unit weight, cohesion, friction angle, elastic modulus, Poisson's ratio, hardness and dilatancy angle, are assigned according to the integer value. The slope cross-section used in this study has dimensions of 1465 m x 322 m in the horizontal and vertical directions, respectively. A pixel of the slope image represents the 2.86 m x 2.86 m in actual dimension. Hence, to represent the actual dimensions of the slope, the generated mesh is rescaled to the actual dimension of the slope. The slope is constrained from horizontal motion along its left and right edges. Along its bottom edge, fully fixed conditions are applied. In order to perform stress-strain analysis for slope stability simulation, the constructed mesh with associated material properties is introduced to the SBFEM elasto-plastic model, applying the gravity load in a single increment (Griffiths & Lane, 1999). Following the shear strength reduction technique, the SRF value in Eq. (29) and Eq. (30) are iteratively increased, weakening the material strength until the model no longer converges for a given number of ceiling iterations. During slope failure, the plastic strain develops from the toe to the crest of the slope, which eventually evolves into a failure plane. For every SRF value, the calculated stress-strain behaviour of the converged model is passed to the Paraview open-source software package (Ahrens et al., 2005) to visualise the results.

Fig. 9. Deterministic case failure surface generated from (a) ABAQUS and (b) SBFEM

Fig. 9 shows the deformation contour simulated by the SBFEM compared with the finite element simulations using ABAQUS at the failure. It is observed that the deformation contours exhibit both qualitative and quantitative similarity. The maximum depth of the slip surface at the final strength reduction factor computed by the SBFEM model is 115 m below the slope crest whereas the results obtained from ABAQUS is 94.5 m. ABAQUS finite element simulations resulted in a slightly flatter slip surface at the bottom while SBFEM gives a slightly curved

bottom slip surface. The middle line of the yellow contour area was selected as slip surface. The maximum depth of the slip surface and deformation of slope may vary due to convergence tolerance value, iteration ceiling value, element sizes and numerical method. The calculated FoS values calculated from a finite element analysis using ABAQUS and SBFEM are also the same and was found to be 3.4.

The image-based quadtree decomposition technique is also suitable to perform efficient meshing, allowing regions of interest to be meshed using finest elements compared to the remainder of the domain. This can be easily controlled by defining regions of interest in the image and controlling the input maximum and minimum cell sizes and the thresholds during quadtree decomposition. In a slope stability analysis, the region along the slope usually exhibits significant deformation and stress gradients. To better capture these effects in the simulations, this region can be meshed with a finer mesh compared with the regions away from the slope. To illustrate this capacity, the deterministic case of the Yallourn slope is again considered. The region considered for mesh refinement is shown in Fig. 10a and is meshed with finer element of size 2^0 while meshing other areas are attempted with coarser element sizes $2^0:2^2$. The generated meshing is shown in Fig. 10b.

Fig. 10. Mesh refinement of the region of interest (a) Region of interest (b) Generated mesh with region of interest

The calculated FoS value is 3.4 and the maximum depth of slip surface is 115m from the crest. It is clear that there is no significant difference with the initial results indicating that the choice of the element sizes used to discretise the mesh in this case study leads to convergence of the results. Fig. 11 shows the deformation and plastic strain contour maps of the failure surface.

Fig. 11. Increased mesh density around the slope area of the deterministic slope (a) X direction displacement (b) plastic strain distribution

4.2. Slope Stability analysis incorporating spatial variation of material properties

In this case study, the ability of SBFEM to robustly integrate spatial variation of the strength parameters, namely the cohesion and friction angle, is demonstrated through image-based mesh generation in combination with Monte Carlo simulations.

Material properties such as cohesion, friction angle and density within a slope exhibit innate variation from point to point due to depositional and geological processes that vary as a function of distance (Lacasse & Nadim, 1996). The material parameters required for large-scale geotechnical design are mainly collected from limited core-samples due to economic and time constraints. To account for uncertainties in the variation of location-dependent materials, random field theory was initially developed by Vanmarcke (1977) to model material variation where physical samples are not available. A case study on the influence of the spatial variation of the material properties in a case study of the Yallourn mine was recently reported (Dyson & Tolooiyan, 2019a). The same spatial maps (images) of material properties generated in (Dyson & Tolooiyan, 2019a) were used in this study to validate the developed technique. The spatial correlation lengths, the horizontal correlation length θ_x and vertical correlation length θ_y defining the fluctuation between any two points in random field for the shear strength parameters of each material are listed in Table 2.

1 **Table 2.** Shear strength correlation length scales

Scale of fluctuation (m)	Coal		Non-coal	
	c	ϕ	c	ϕ
θ_x	320.3	772.4	812	627.8
θ_y	23.1	8.7	35	13.1

2

3 The quadtree meshing technique as explained in Section 2.2 is applied to generate the mesh (see Fig. 14)

4 considering the three images representing the stratigraphy (see Fig. 7), the spatial variation of the cohesion (see

5 Fig. 12) and the spatial variation of the friction angle (see Fig. 13). The mesh is generated by specifying a

6 minimum and maximum cell sizes of $2^0;2^3$.

13 **Fig.12.** Spatial variation instance for the cohesion

14 (kPa) cross section of the Yallourn mine

13 **Fig. 13.** Spatial variation instance of the friction angle

14 (degree) cross section of the Yallourn mine

16 **Fig. 14.** Generated computational mesh

21 The validation of the developed technique was performed using 40 samples of randomly generated spatial

22 variations of the cohesion and friction angle. After mesh generation, the FoS of the slopes is then calculated for

each of the samples. The results obtained from the present study is compared to those obtained from the previous RFEM study (Dyson & Tolooiyan, 2020).

Fig. 15 presents a histogram comparing the FoS distribution computed from the SBFEM and the RFEM results for 40 samples. The results of the SBFEM Factor of Safety distribution shows good agreement with those obtained through previous RFEM simulation. For the RFEM simulations, the mesh consisted of 40,000 elements whereas the SBFEM mesh contains on average 7,850 elements. Moreover, mesh generation in RFEM using Abaqus is not straightforward unlike the quadtree image-based decomposition as outlined in Section 2. The mesh generation and material properties assigned in the mesh process in the SBFEM is fully automatic and the biggest advantage of this method is the ability to create many different models automatically as opposed to Abaqus, where the mesh generation requires significant user input as well as manual handling to map the random field parameters to the finite element mesh.

Fig. 15. RFEM and SBFEM after 40 simulations

To compare the slope performance of individual random field instances, eight sample cases of random spatial variation are identified with their associated safety factors are shown in Table 3. It is noted that calculated FoS

values for the eight RFEM cases match well with the results generated from SBFEM. The FoS values vary by ± 0.1 for each of the eight cases. Due to the simplicity of implementing image-based random field models, with comparable results to RFEM, the applicability of spatially variable SBFEM slope stability analysis is evident.

Table 3. FoS value comparison between SBFEM and RFEM (for selected samples)

Sample No	SBFEM	RFEM
1	2.4	2.5
2	2.7	2.7
3	2.7	2.6
4	3.2	3.1
5	3.0	2.9
6	2.9	3
7	3.0	2.9
8	2.8	2.8

Fig. 16 compares the slip surfaces generated from three different samples using the RFEM and the SBFEM. When considering the sample cases 2, 6 and 10 failure surfaces, the RFEM and SBFEM simulations result in an almost flat bottom slip surface.

In addition to the comparison of final failure surfaces, the maximum depths of the slip surfaces are compared for selected cases, as shown in Table 4.

Table 4. Maximum depth of the slip surface from the crest of the slope (for selected samples)

	RFEM	SBFEM
Case No	Max. Depth(m)	Max. Depth(m)
1	-99	-104
2	-87	-95
3	-96	-95
4	-97	-90
5	-108	-101
6	-90	-98
7	-115	-109
8	-91	-94

Further investigation is conducted for a slope with spatial variable, by considering a finer mesh in the region surrounding the slope as discussed in Section 4.1. The region selected for mesh refinement is discretised with a cell size of 2^0 while the remaining region is meshed by specifying setting the ratio of the minimum and maximum cell size to $2^0:2^3$. The three digital images consisting of the stratigraphy of the soil image, the spatial variation of the soil cohesion image and the spatial variation of the soil friction angle image are utilised to generate the computational mesh as discussed in Section 2.2 while applying mesh refinement in the region of interest. The calculated FoS value is 2.6 which agrees well with the FoS calculated with RFEM. It is clear that there is no

significant difference in the failure surface indicating that the mesh adopted is of a sufficiently fine. The deformation contours and the plastic strain contours of the failure surface are shown in Fig. 17.

Fig. 17. Increased mesh density around the slope area of the spatial variable slope (a) X direction displacement (b) plastic strain distribution

4.3. Change in stratigraphy geological and slope geometry for an existing model

In this section, the flexibility of altering the geological and the geometrical features for excavation and backfilling using image-based analyses are demonstrated. The simulation of several modifications to slope geometry and geology are presented, focusing on the removal of geometric elements (slope excavation) and the addition of material (slope backfilling). Compared with the finite element method, such types of modification often require significant effort in human interaction, pre-processing and remeshing to consider changes to an initial slope model. Four scenarios are considered in this study: adding a previously undetected hypothetical thin coal layer to the deterministic slope case to highlight the simplified process of geological feature modification; an excavation case where the slope is arbitrarily excavated; surcharge filling at the toe to increase the stability of slope and backfilling of a complete slope are considered. These scenarios are shown in Fig.18. The modified cases are easily created in any image editing software, then are reintroduced following the procedures detailed in the Fig. 5 flow chart. This demonstrates the efficiency of the image-based SBFEM method to simulate engineering solutions into pre-existing models. In these examples, spatial variation is not considered.

Fig. 18. Alteration of geological and geometry data in an existing model (a) Addition of a thin interseam geological features to the slope (b) Excavation scenario of the working face (c) Surcharge filling to increase FoS (d) Backfilling and final slope establishment

4.3.1. Results generated for alternation of geological and geometry data in an existing model

Fig. 19. X displacement contour maps of final failure surfaces of (a) Adding thin interseam geological features to the slope (b) Excavation scenario of the working face (c) Surcharge filling to increase FoS (d) Backfilling and final slope establishment cases

Fig. 20. Plastic strain distribution contour maps of final failure surfaces of (a) Addition of a thin interseam geological features to the slope (b) Excavation scenario of the working face (c) Surcharge filling to increase FoS (d) Backfilling and final slope establishment cases

Fig. 19 shows the resulting failure surfaces after alteration of geological and geometry data to the existing model. The calculated FoS values are 3.0, 1.8, 3.7 and 3.4 respectively for each of the cases. Fig. 20 shows the plastic strain distribution of the final failure surfaces for all the cases shown in Fig. 18.

It is observed that the addition of a thin interseam layer reduces the FoS of the slope. Since coal is weaker and more compliant than soil, the FoS reduces compared because the domain now contains a larger content of a weaker material compared with its predecessor. For the case where some excavation work has taken place, the FoS also reduces. Excavation increases the slope angle and results in a more unstable slope. The result is compliant with observations in practice (Salmasi et al., 2019). Addition of surcharge at the toe or backfilling of the slope helps to stabilise the slope, resulting in increased FoS. This result also conforms to what is observed in engineering practice where backfilling and surcharging are employed during mine rehabilitation to increase/enhance the stability of slopes (Yan et al., 2013).

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Discussion and conclusion

In this study, an alternative numerical method is developed, utilising the scaled boundary finite element method in combination with the image-based quadtree decomposition technique to perform slope stability analysis in the context of shear strength reduction technique. The scaled boundary finite element method can be formulated on elements with arbitrary shapes and sizes and is used in combination with an image-based input through the quadtree decomposition technique. An elasto-plastic model was developed with the scaled boundary finite element method, integrating with the Mohr-Coulomb criterion, while the return mapping algorithm, coupled with the Newton-Rapson method, has been utilised to project the closest stress increment in the elasto-plastic model. The image-based quadtree decomposition method is utilised to generate a geometrical mesh for a given slope image. The strength reduction technique is applied for the developed scaled boundary finite element elasto-plastic model to assess the slope stability analysis.

The quadtree decomposition technique is a fully automatic method that reduces the burden and complexity of meshing, human involvement and allows the capture of complicated features and material types denoted by pixel information of an image. These combined features allow spatially variable strength parameters to be implemented through spatially variable parameter distribution images. Since the image-based quadtree mesh can be generated concurrently considering multiple images, a range of material properties of a slope can be easily overlapped and assigned to a single computational mesh that allows consideration of a combination of spatially variable material parameters in a single model. Selective meshing of regions of interest can be easily implemented allowing mesh refinement in these regions to be performed automatically. Furthermore, the image-based method makes it significantly easier to model geometrical changes in the model simply by editing an image.

Three case studies of applications are provided: the first being a deterministic case of the Yallourn mine, the second being the consideration of spatially variable strength parameters and the third being altered geometrical

and geological data, demonstrating the slope stability analysis process with image-based SBFEM method and its flexibilities to alter data over time. The factor of safety value calculated for the first case matches the ABAQUS result, reflecting that the presented method generates results comparative to the finite element method. The consideration of the spatial variation of strength parameters through the presented method is quick and easy compared to the RFEM method as it automatically captures the material types and spatial variation parameters while also generating the computational mesh. It is clearly demonstrated through the second case and the generated results match the results of the RFEM method. The depth and the shape of the potential failure surface for spatial variation cases are compared with SBFEM and RFEM models, with the maximum failure depths and shape of failure surfaces are noted to be in agreement. The changes to geometry are introduced into existing images demonstrating the speed of alteration, as presented in the third case. As highlighted by the results shown in this study, the image-based Scaled Boundary Finite Element Method is a beneficial tool in modelling a range of complexities and challenges common to large-scale open-pit mine slope stability analysis.

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