4.1 Introduction

The ionosphere is part of the earth’s upper atmosphere and located at a height of between 50-1000km above the surface. Its high spatial and temporal variability has a significant effect on GPS signals travelling from the satellites to the receivers. Moreover, the condition of the ionosphere is strongly related to the 11-year solar cycle. Hence, in a local deformation monitoring network the ionospheric effect has to be accounted for, even for short baselines, and especially in equatorial regions.

This chapter describes the ionosphere and its effects on the GPS signals. A procedure to adjust single-frequency GPS observations for the ionospheric delay effects using empirical corrections derived from a dual-frequency network of reference receivers is described. The nature of these correction terms is investigated by analysing GPS data collected over baselines of varying length, in different geographical locations, and at different periods of sunspot activity (and hence varying ionospheric conditions).

4.2 Sunspot Number and the Solar Cycle

Sunspots are the physical expression of complex magneto-dynamical processes within the Sun and are seen as dark areas on the solar disk that wax and wane (NASA, 2002). Sunspots are darker than their surrounding area because they are cooler than the average temperature of the solar surface, and can therefore easily be observed. The pattern of sunspots on the Sun varies on timescales from a few hours to many years. An index called the sunspot number was introduced approximately 300 years ago and has since continued to be used to quantify the abundance of spots.
The sunspot number (here denoted SSN) is defined as (IPS, 2002):

$$SSN = K (10 \cdot G + I)$$  \hspace{1cm} (4-1)

where $G$ is the number of sunspot groups visible on the Sun; $I$ is the total number of individual spots; and $K$ is an instrumental factor to take into account differences between observers and observatories.

In this context it is necessary to clarify a few terms associated with solar activity. As explained in Knight (2000), sudden increases in the intensity of solar radiation associated with sunspot activity are known as solar flares. Solar winds are particles charged with high energy that are emitted from the Sun. Coronal holes are low density regions of the solar corona that are associated with these solar winds. Geomagnetic storms are large variations in the strength and direction of the earth’s magnetic field caused by eruptions on the Sun that eject a mixture of electrons, protons and ions into the solar wind.

The solar cycle has an average length of 11 years. However, cycles vary considerably in length from as short as 9 years up to almost 14 years (IPS, 2002). Due to its large day-to-day variability the sunspot number is usually averaged over a month. If smoothed over a 13-month period, the SSN effectively charts the progress of the solar cycle. The daily and monthly averages exhibit considerable variation with respect to the smoothed curve due to bursts of rapid solar region growth often associated with events like solar flares. Figure 4.1 shows the monthly sunspot numbers for the entire historical record (top) and for the most recent solar cycle 23 (bottom). The sunspot data are freely available on the internet (NASA, 2002; IPS, 2002). In the bottom graph the high-frequency curve depicts the observed monthly sunspot number, while the low-frequency curve shows the smoothed (solid line) and predicted (dashed line) monthly values. The most recent solar maximum occurred in 2000; however, the ionospheric activity remains high for several years after the solar maximum.
4.3 The Ionosphere

The ionosphere is a band of the atmosphere located approximately 50-1000km above the earth’s surface, thinning into the plasmasphere (or protonosphere) and eventually into the interplanetary plasma at greater heights. Most of the ionosphere is electrically neutral, but ionisation results when solar radiation strikes the ionosphere. The upper atmosphere then becomes an electrical conductor, which supports the flow of electric currents, and hence affects the propagation of radio waves. The condition of the ionosphere is strongly related to the 11-year solar cycle (see previous section).

The ionosphere is traditionally divided into several regions (D, E and F) and layers, on the basis of the level of ionisation within a region (Anderson & Fuller-Rowell, 1999; Goodman & Aarons, 1990). Figure 4.2 shows the ionospheric layers and the principle ions that compose each region; the electron density is also included (in units of electrons/cm$^3$). The F2 layer is particularly important for GPS because here the electron concentrations reach their highest values. The effect of the ionosphere on GPS signals travelling from the satellites to the receivers is dependent on the electron content along
the signal path and the frequency of the signal (Seeber, 1993). Other factors influencing the ionospheric refraction are geographic location, the period in the solar cycle and time of day.

![Regions and layers of the ionosphere](image.png)

**Fig. 4.2:** Regions and layers of the ionosphere, their predominant ion populations and electron density (Anderson & Fuller-Rowell, 1999)

The propagation speed \( v \) of a signal is related to the refractive index \( n \) according to (e.g. Hofmann-Wellenhof et al., 2001; Rizos, 1997):

\[
v = \frac{c}{n}
\]  

(4-2)

where \( c \) is the speed of electromagnetic radiation in a vacuum. Note that the refractive index is dimensionless.
Seeber (1993) gives a useful approximate expression for the refraction coefficient $n_p$ for carrier phase observations:

$$n_p = 1 - 40.3 \cdot \frac{n_e}{f^2}$$  \hspace{1cm} (4-3)

where $n_e = \text{electron density integrated along the signal propagation path \text{[electrons/m}^3\text{]}}$

$$f = \text{carrier frequency \text{[MHz]}}$$

Note that the coefficient 40.3 contains several constant parameters including their dimensions. The refraction coefficient for the pseudorange measurement $n_g$ is of the same size but opposite sign:

$$n_g = 1 + 40.3 \cdot \frac{n_e}{f^2}$$  \hspace{1cm} (4-4)

This indicates that measured pseudoranges are ‘too long’ compared to the geometric distance between satellite and receiver, while carrier phase observations are ‘too short’. The terms *group delay* and *phase advance* are also used in this context (Kleusberg & Teunissen, 1996). Note that $n_g$ is expressed as a group index (a wave group generated by superposition of different waves of different frequencies), as opposed to the phase index $n_p$ of a particular wave with constant wavelength. Note also that the electron density $n_e$ must be known in order to determine the refraction indices.

Integrating the electron density along the signal path results in the widely used TEC measure (Total Electron Content), expressed in TEC units (TECU), with 1 TECU corresponding to $10^{16}$ free electrons per m$^2$. TEC is highly variable on both a temporal and spatial basis. However, the dominant variability is diurnal. TEC is a function of many variables, including long and short term fluctuations in solar radiation, magnetic activity, season, time of day, user location and viewing direction (Klobuchar, 1987). Generally, the equivalent vertical TEC (VTEC) is determined by dividing the slant TEC by the secant of the zenith angle at a mean ionospheric height (usually between 350 and 400km) in order to permit convenient comparison between different data sets. Typical daytime values of VTEC for mid-latitude sites are of the order of $10^{18}$ electrons/m$^2$ with corresponding nighttime values of $10^{17}$ electrons/m$^2$. However, these values can be
exceeded by a factor of two or more, especially in near-equatorial regions (Kleusberg & Teunissen, 1996). Values of TEC from $10^{16}$ to $10^{19}$ electrons/m$^2$ represent the extremes observed in the earth’s ionosphere (Klobuchar, 1996).

The ionosphere is a dispersive medium for microwaves, i.e. the refractivity depends on the frequency of the propagating signal. Hence, measurements on both the L1 and L2 frequencies can be used to account for the ionospheric effect on GPS observations (see section 4.4).

The ionosphere is most active in a band extending up to approximately 20° on either side of the geomagnetic equator (Fig. 4.3). This is also one of the two regions where small-scale ionospheric disturbances (scintillations) mainly occur. The other being the high-latitude (auroral) regions close to the poles.

Scintillations are short-term variations in the amplitude and phase of radio signals travelling through the ionosphere. While auroral and polar scintillations are mainly the result of geomagnetic storms that are associated with solar flares and coronal holes, in the equatorial region scintillations are caused by irregularities in the F-layer of the ionosphere following the passage of the ‘evening terminator’, the boundary that divides day from night (Knight, 2000). They generally occur between approximately one hour
after sunset until midnight (Klobuchar, 1996) and should have disappeared by 3am local time (IPS, 2002). Figure 4.4 illustrates the L-band ionospheric scintillation fading depths for solar minimum and maximum conditions. Note that the main anomaly region located at ±15° of the geomagnetic equator experiences the deepest signal fades of up to 20dB below the mean signal level. Less intense fading is experienced at the geomagnetic equator and in regions surrounding the main anomaly region.

![Fig. 4.4: Ionospheric scintillation during high and low solar activity](Goodman & Aarons, 1990)

The occurrence of scintillations also varies with the seasons. Between April and August they are less severe in the American, African and Indian longitude regions, but are at a maximum in the Pacific region, while the situation is reversed from September to March (Seeber, 1993). Using data collected at a site located very close to the geomagnetic equator in Peru over three and a half years from January 1997, Doherty et al. (2000) studied the seasonal dependence of scintillation activity in the equatorial region. Their study revealed virtually no trace of scintillations in the months between June and August, while peak occurrence could be observed in the equinox seasons. A clear increase in scintillation effects with increasing sunspot activity (approaching a solar maximum) was also apparent.

High-latitude scintillations are strongly dependent on geomagnetic activity levels, but can occur in all seasons and are not limited to the local nighttime hours (Essex et al., 2001; Doherty et al., 2000). In mid-latitudes scintillations are rarely observed, but Medium-Scale Travelling Ionospheric Disturbances (MSTIDs) occur frequently, mainly in the
daytime in the winter months, during periods of high solar activity, with a maximum
around local noon (Wanninger, 1999). In all regions, however, increased solar activity
amplifies scintillation frequency and intensity. Numerous researchers have investigated
the nature of the scintillations and their impact on GPS observations. The results of these
studies can readily be found in the literature (e.g. Dodson et al., 2001; Fedrizzi et al.,
2001; Fu et al., 1999; Knight et al., 1999; Nichols et al., 1999).

The effect of the ionosphere on GPS coordinate solutions can be very severe, even on
short baselines. Wanninger (1993) reports that single-frequency data from a 10km
baseline located in southern Brazil, which was processed in data blocks of one hour,
showed coordinate errors of 1cm (or 1ppm of the baseline length) before sunrise, of up to
5cm (5ppm) during the daylight hours, and more than 30cm (30ppm) between sunset and
midnight.

With the removal of Selective Availability in May 2000, the ionosphere is now the
largest individual systematic error in the GPS error budget, accounting for as much as
80% or more (Kunches & Klobuchar, 2001). The ionospheric range error on L1 in the
zenith direction can reach 30m or more, and near the horizon this effect is amplified by a
factor of about three (Kleusberg & Teunissen, 1996).

4.4 Ionospheric Corrections for Dual-Frequency Users

Due to the dispersive nature of the ionosphere, a linear combination of the L1 and L2
measurements can be formed to correct for the ionospheric delay. In the case of
pseudorange measurements, the ionospheric bias can be estimated and subsequentially
removed from the L1 measurements by forming the linear combination (Kleusberg &
Teunissen, 1996):

\[
d_{\text{ion}} = \frac{f_2^2}{f_2^2 - f_1^2} \cdot \[P_1 - P_2\] 
\]

(4-5)

where \( f_1 \) and \( f_2 \) are the L1 and L2 carrier frequencies respectively, and \( P_1 \) and \( P_2 \) are the
L1 and L2 pseudorange measurements.

Similarly, carrier phase measurements can be corrected using:
where $\phi_1$ and $\phi_2$ are the L1 and L2 carrier phase measurements (in units of length) respectively, $\lambda_1$ and $\lambda_2$ are the L1 and L2 carrier wavelengths, and $N_1$ and $N_2$ are the L1 and L2 integer cycle ambiguities. The absolute values of $N_1$ and $N_2$ cannot be determined, but they remain constant for continuous observations without cycle slips. Hence the variation in the ionospheric delay, also called the differential ionospheric delay, can be obtained.

The ionospheric effect on carrier phase observations can be eliminated by forming the so-called L3 or ionosphere-free linear combination (e.g. Leick, 1995; Hofmann-Wellenhof et al., 2001; Rizos, 1997):

$$
\phi_3 = \frac{f_1^2}{f_1^2 - f_2^2} \cdot \phi_1 - \frac{f_1 f_2}{f_1^2 - f_2^2} \cdot \phi_2
$$  \hfill (4-7)

For double-differenced observables, the above equation can be written as (Rizos, 1997):

$$
\nabla \Delta \phi_{1, \text{iono-free}} = \nabla \Delta \rho + \frac{f_1^2}{f_1^2 - f_2^2} \cdot \nabla \Delta N_1 - \frac{f_1 f_2}{f_1^2 - f_2^2} \cdot \nabla \Delta N_2
$$  \hfill (4-8)

Similarly, a pseudorange L3 linear combination can also be formed (Ibid, 1997). However, measurements on both frequencies are obviously needed to determine these corrections.

The Center for Orbit Determination in Europe (CODE), located at the University of Berne in Switzerland, generates daily global ionosphere maps (GIMs) based on data collected at global IGS sites. Schäer et al. (1996) describe the procedure as follows. The geometry-free linear combination of double-differenced carrier phase observations made by the IGS network are processed to extract the global TEC information. Data are processed with a 3-minute observation rate using an elevation cut-off angle of 20°. The global TEC distribution is represented over 24 hours by spherical harmonics up to degree
8 in a geographical reference frame, which is rotating with the mean Sun. A spherical ionospheric shell at a height of 400km above the earth’s mean surface is adopted. Figure 4.5 shows an example of a global ionosphere map. While the TEC values range from 2-12 at a latitude of 45°N, they show much larger variations of between 2-34 at the equator. Also note the steep gradients in TEC just after sunset (90° sun-fixed longitude) and around midnight (180° sun-fixed longitude) in the equatorial region.

![Global Ionosphere Map (GIM)](image)

Fig. 4.5: Global Ionosphere Map (GIM) for day 073, 1996 (Schaer et al., 1996)

These maps are available on the internet (AIUB, 2002). However, being global maps, they are not very effective in modelling the ionospheric conditions in local GPS networks for short observation periods. Moreover, even though there are a large number of IGS sites, they are unevenly distributed, with most of the GPS stations being situated in the mid-latitude region of the northern hemisphere. The smaller number of GPS receivers in the equatorial region and the southern hemisphere, and consequently the reduced number of available TEC measurements, results in the ionospheric modelling to be less accurate for these regions.

### 4.5 A Mixed-Mode GPS Network Processing Approach to Account for the Ionospheric Effect
In general it is assumed that short baselines (say up to 10km in length) are not severely affected by ionospheric and tropospheric delays, as the GPS signals propagate through essentially the same portion of the atmosphere. Hence the bias effects will be very similar at either end of the baseline, and therefore effectively cancel in between-receiver data differencing, which is part of the double-differencing process. However, the geographical location in the equatorial region, and the significant height differences of 1400m between single-frequency receivers on an Indonesian volcano, make such assumptions questionable, even for short baselines.

While data from dual-frequency GPS receivers can account for the ionospheric delay directly (by the appropriate linear combination of measurements made on both frequencies – section 4.4), data from single-frequency receivers cannot be corrected in this way. Single-frequency GPS data processing can use a simple ionosphere model transmitted within the navigation message to account for about 50% RMS of the ionospheric range delay (see Klobuchar, 1987, for details of the model). However, this is not sufficient for volcano deformation monitoring applications where the objective is to detect movements of a few centimetres or less. Moreover, it cannot model the highly variable (in location and magnitude) TEC behaviour in equatorial regions, where most of the world’s volcanoes are located.

In order to optimise the single-frequency volcano deformation monitoring system described in chapter 2, a network of three dual-frequency GPS receivers surrounding the deformation zone has been proposed. In this mixed-mode GPS network approach, the dual-frequency receivers would be used to generate ‘correction terms’, which can then be applied to the single-frequency observations to account for the ionospheric biases. Over the last few years several methods have been developed to generate ionospheric corrections based on a network of reference stations (e.g. Camargo et al., 2000; Chen et al., 2000; Odijk et al., 2000; Rocken et al., 2000; Yuan & Ou, 2001).

As described in Han (1997), such a fiducial network should ideally surround the inner single-frequency network (see Fig. 2.13). The fiducial reference stations operate continuously in order to determine a regional ionosphere model on an epoch-by-epoch
and satellite-by-satellite basis, as proposed by Wanninger (1995). A single layer ionospheric model at a height of typically 350km is used, where it is assumed that the entire ionospheric layer is compressed into an infinitesimally thin surface (Fig. 4.6). In the case of a relatively small fiducial network, with baseline lengths of no more than 80km, the slant ionospheric delay measured at the reference stations can be considered to be in the zenith direction. The variation in the ionospheric delay across the network is also assumed to be linear. It should be noted that this assumption may not be valid in periods of increased solar activity due to the highly variable (in space and time) scintillation effects.

At every epoch the ionospheric delay for the L1 signal can be determined relative to the reference receiver and the reference satellite (Han & Rizos, 1996d):

\[
I_1^i(k) = \frac{f_2^2}{f_1^2 - f_2^2} \cdot \left( (\Delta V \phi_{L1}^{ij}(k) - \lambda_1 \cdot \Delta V N_{L1}^{ij}) - (\Delta V \phi_{L2}^{ij}(k) - \lambda_2 \cdot \Delta V N_{L2}^{ij}) \right)
\]  

(4-9)

where \(\Delta V \phi_{L1}^{ij}(k), \Delta V \phi_{L2}^{ij}(k)\) are the double-differenced observables of the L1 and L2 carrier phase (in units of metres) relative to the reference receiver and the reference satellite. Note that for a rising satellite, no ionospheric corrections can be applied until the ambiguities for that satellite have been resolved.
The ionospheric delay can then be expressed as:

\[ I_{ij}^k(k) = x_i \cdot a_{ij}^N(k) + y_i \cdot a_{ij}^E(k) \]  \hspace{1cm} (4-10)

where \( a_{ij}^N(k) \) and \( a_{ij}^E(k) \) represent the north and east slope components for satellite \( j \) relative to the reference satellite and the reference station at epoch \( k \). The slope parameters are unique to each satellite/receiver pair and change over time as the satellites move across the sky. The parameters \( x_i \) and \( y_i \) refer to the relative location of the reference station \( i \).

This correction approach linearly interpolates the TEC across a surface and approximates the ionospheric delay at any point within the fiducial triangle. However, it cannot entirely account for scintillation effects, which can be highly localised.

The ionospheric correction method described above is part of the linear combination model proposed by Han & Rizos (1996d) and Han (1997). This model can account for orbit bias and ionospheric delay, as well as mitigate tropospheric delay, multipath and measurement noise across the network. Thus, data from the fiducial GPS reference station network can be used to derive empirical corrections to the double-differenced carrier phase data formed between the stations of the inner network. The procedure was investigated by Chen (2001) and Chen et al. (2001a), and is described in this section.

### 4.5.1 Single-Differenced Model

The single-differenced carrier phase observation can be written as (Han, 1997):

\[ \Delta \phi = \Delta \rho + \Delta \rho - c \cdot \Delta T + \lambda \cdot \Delta N - \Delta d_{\text{ion}} + \Delta d_{\text{trop}} + \Delta d_{\text{mp}} + \epsilon \Delta \phi \]  \hspace{1cm} (4-11)

where
- \( \Delta \) = single-difference operator (difference between user and reference receiver)
- \( \phi \) = carrier phase observation in units of metres
- \( \rho \) = distance between receiver station and satellite
If more than one reference station is available, several single-differenced carrier phase observables can be obtained between the user receiver and the reference receivers. In order to account for the distance dependent biases (orbit error, ionospheric delay and tropospheric delay) in equation (4-11), weights are introduced. These weights are inversely proportional to the baseline lengths and are used to average out the different distances between the user and the reference stations. This idea was suggested by Wu (1994) and Han & Rizos (1996d). In the case of a fiducial network consisting of three reference stations, a set of parameters $\alpha_i$ can be determined such that the following conditions are satisfied:

$$\sum_{i=1}^{3} \alpha_i \cdot (\tilde{X}_u - \tilde{X}_i) = 0$$

(4-12)

where $\tilde{X}_u$ and $\tilde{X}_i$ are the coordinate vectors of the user receiver and reference receiver in the Gaussian plane coordinate system respectively, and $\alpha_i$ is the weight for reference station i.

To average out the different distances between the user receiver and the reference receivers, the sum of the weights is required to be equal to 1:

$$\sum_{i=1}^{3} \alpha_i = 1$$

(4-13)
In order to minimise the standard deviation of the linear combination of single-differenced observations, another constraint should be added:

$$\sum_{i=1}^{3} \alpha_i^2 = \min$$  \hspace{1cm} (4-14)

The linear combination of the single-differenced observables can now be written as:

$$\sum_{i=1}^{3} \alpha_i \cdot \Delta \phi_i = \sum_{i=1}^{3} \alpha_i \cdot \Delta \rho_i + \sum_{i=1}^{3} \alpha_i \cdot \Delta d_{\text{mp},i} + \sum_{i=1}^{3} \alpha_i \cdot \Delta d_{\text{trop},i} + \sum_{i=1}^{3} \alpha_i \cdot \Delta d_{\text{ion},i} + \sum_{i=1}^{3} \alpha_i \cdot \Delta N_i$$

$$- \sum_{i=1}^{3} \alpha_i \cdot \Delta d_{\text{mp},i} + \sum_{i=1}^{3} \alpha_i \cdot \Delta d_{\text{trop},i} + \sum_{i=1}^{3} \alpha_i \cdot \Delta d_{\text{ion},i} + \sum_{i=1}^{3} \alpha_i \cdot \Delta N_i$$  \hspace{1cm} (4-15)

**Orbit Bias**

According to equation (4-12), the orbit bias term has been shown to be:

$$\sum_{i=1}^{3} \alpha_i \cdot \Delta \rho_i \approx 0$$  \hspace{1cm} (4-16)

**Ionospheric Delay**

The ionospheric delay can be expressed as:

$$\sum_{i=1}^{3} \alpha_i \cdot \Delta d_{\text{ion},i} = d_{\text{ion},1} - d_{\text{ion},3}$$

$$d_{\text{ion},1} = \begin{bmatrix} \alpha_1 & \alpha_2 \end{bmatrix} \begin{bmatrix} d_{\text{ion},1} - d_{\text{ion},3} \\ d_{\text{ion},2} - d_{\text{ion},3} \end{bmatrix}$$  \hspace{1cm} (4-17)

If the ionospheric delay at the user station can be correctly interpolated from the delays determined at the reference stations, this term will be very close to zero. With increasing distance between the reference stations, however, the residual bias will become larger due to errors in the ionospheric delay interpolation.

**Tropospheric Delay**
If the tropospheric delay can be interpolated from the residual tropospheric delay at the reference stations, the tropospheric delay can be represented as:

\[
\sum_{i=1}^{3} \alpha_i \cdot \Delta d_{\text{trop},i} = d_{\text{trop},u} - d_{\text{trop},3} - \left[ \begin{array}{cc} x_u & y_u \\ \end{array} \right] \cdot \left[ \begin{array}{cc} x_1 & y_1 \\ x_2 & y_2 \\ \end{array} \right]^{-1} \cdot \left[ \begin{array}{c} d_{\text{trop},1} - d_{\text{trop},3} \\ d_{\text{trop},2} - d_{\text{trop},3} \end{array} \right]
\]

where \((x_1, y_1), (x_2, y_2)\) and \((x_u, y_u)\) are the coordinates of the reference and user receivers relative to reference station 3 in the Gaussian plane coordinate system. How close this term is to zero depends on the spatial correlation characteristics of the tropospheric delay. The residual tropospheric delay is largely influenced by the wet component of the troposphere, which is highly variable with height, time and geographic location. It can be expected that the tropospheric delay will be mitigated to some extent, but its effectiveness is an unknown function of the distance between the GPS receivers. For local networks, however, it can be assumed that the term is very close to zero.

**Multipath**

The carrier phase multipath effect can be expressed as:

\[
\sum_{i=1}^{3} \alpha_i \cdot \Delta d_{\text{mp},i}^\phi = d_{\text{mp},u}^\phi - \sum_{i=1}^{3} \alpha_i \cdot d_{\text{mp},i}^\phi
\]  

The second term on the right side of equation (4-19) is the weighted mean of the multipath values at the three reference receivers for this satellite. Due to the random nature of multipath at different receivers, the weighted mean value will be reduced if all \(\alpha_i\) \((i = 1, 2, 3)\) are positive and less than 1, although the weight \(\alpha_i\) is not derived from its standard deviation. On the other hand, the multipath at the user receiver will become a high-frequency bias, and will be assumed to be close to random noise (Zhang & Schwarz, 1996). Therefore, the multipath term can be assumed to have been significantly reduced and will consequently be ignored in the functional model.

**Observation Noise**
The standard deviation of the one-way carrier phase observation can be approximated as a function of the elevation angle. If all GPS network receivers are located within a region of about 100km radius, the elevation of a satellite is approximately the same. If $\sigma_j$ denotes the standard deviation of a one-way carrier phase observation, the standard deviation of the linear combination of single-differenced observations can be represented as:

$$\sigma_j = \sqrt{1 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2} \cdot \sigma_j$$  (4-20)

In comparison to the standard deviation of the single-differenced carrier phase observation $\sqrt{2} \cdot \sigma_j$, the standard deviation will become smaller if the user receiver is located inside the triangle formed by the reference stations. Hence, the overall noise across the network will be reduced.

The single-differenced functional model for the linear combination (equation 4-15) can now be written in a simplified form:

$$\sum_{i=1}^{3} \alpha_i \cdot \Delta \phi_i = \sum_{i=1}^{3} \alpha_i \cdot \Delta \rho_i - c \cdot \sum_{i=1}^{3} \alpha_i \cdot \Delta dT_i + \lambda \cdot \sum_{i=1}^{3} \alpha_i \cdot \Delta N_i + \epsilon \sum_{i=1}^{3} \alpha_i \cdot \Delta \phi_i$$  (4-21)

Considering a user receiver $u$ and the relation (Han, 1997):

$$\sum_{i=1}^{3} \alpha_i \cdot \Delta \phi_i = (\phi_u - \phi_3) - [\alpha_1 \cdot (\phi_1 - \phi_2) + \alpha_2 \cdot (\phi_2 - \phi_3)]$$

$$= \Delta \phi_{u,3} - \left[\alpha_1 \cdot \Delta \phi_{1,3} + \alpha_2 \cdot \Delta \phi_{2,3}\right]$$  (4-22)

equation (4-21) can be expressed as:

$$\Delta \phi_{u,3} - \left[\alpha_1 \cdot \Delta \phi_{1,3} + \alpha_2 \cdot \Delta \phi_{2,3}\right] = \Delta \rho_{u,3} - \left[\alpha_1 \cdot \Delta \rho_{1,3} + \alpha_2 \cdot \Delta \rho_{2,3}\right] - c \cdot \sum_{i=1}^{3} \alpha_i \cdot \Delta dT_i$$

$$+ \lambda \cdot \Delta N_{u,3} - \left[\alpha_1 \cdot \Delta N_{1,3} + \alpha_2 \cdot \Delta N_{2,3}\right] + \epsilon \sum_{i=1}^{3} \alpha_i \cdot \Delta \phi_i$$  (4-23)

4.5.2 Double-Differenced Model
The double-differenced carrier phase observable can be written as (Chen et al., 1999):

\[
\nabla \Delta \phi = \nabla \Delta \rho + \nabla \Delta dp + \lambda \cdot \nabla \Delta N - \nabla \Delta d_{\text{ion}} + \nabla \Delta d_{\text{trop}} + \nabla \Delta d_{\text{rep}} + \epsilon_{\nabla \Delta \phi}
\]  

(4-24)

where \( \nabla \Delta \) denotes the double-differencing operator. Note that the receiver clock error term present in the single-differenced observable cancels by forming the between-satellite differences.

Based on the discussion in the last section and considering equation (4-23), the double-differenced observation model can also be represented as:

\[
\nabla \Delta \phi_{u,3} - \left[ \alpha_1 \cdot \nabla \Delta \phi_{1,3} + \alpha_2 \cdot \nabla \Delta \phi_{2,3} \right] = \nabla \Delta \rho_{u,3} - \left[ \alpha_1 \cdot \nabla \Delta \rho_{1,3} + \alpha_2 \cdot \nabla \Delta \rho_{2,3} \right] + \lambda \cdot \nabla \Delta N_{u,3} - \left[ \alpha_1 \cdot \nabla \Delta N_{1,3} + \alpha_2 \cdot \nabla \Delta N_{2,3} \right] + \epsilon_{\nabla \Delta \phi_i} 
\]

(4-25)

Residual vectors can be formed from the double-differenced observations between reference stations 1 & 3 and 2 & 3:

\[
V_{1,3} = \nabla \Delta \phi_{1,3} - \nabla \Delta N_{1,3} - \nabla \Delta \rho_{1,3}
\]

(4-26a)

\[
V_{2,3} = \nabla \Delta \phi_{2,3} - \nabla \Delta N_{2,3} - \nabla \Delta \rho_{2,3}
\]

(4-26b)

The double-differenced observable can then be written as:

\[
\nabla \Delta \phi_{u,3} - \left[ \alpha_1 \cdot V_{1,3} + \alpha_2 \cdot V_{2,3} \right] = \nabla \Delta \rho_{u,3} + \lambda \cdot \nabla \Delta N_{u,3} + \epsilon_{\nabla \Delta \phi_i} 
\]

(4-27)

After the initialisation of the reference stations, the integer ambiguities are known, and together with the known coordinates the correction vectors \( V_{1,3} \) and \( V_{2,3} \) can be determined. The correction term \( \left[ \alpha_1 \cdot V_{1,3} + \alpha_2 \cdot V_{2,3} \right] \) can now be obtained and applied to the user receiver data.

4.5.3 Extended Model for Multiple User Receivers
The model described above can be extended to account for two or more user receivers situated within the fiducial network of reference receivers. This will be the case in a mixed-mode GPS-based volcano deformation monitoring network, comprising three dual-frequency reference receivers surrounding the single-frequency receivers located in the deformation zone on the volcano. Figure 4.7 depicts such a scenario, triangles and dots denoting GPS reference sites and user sites respectively.

Consider two user stations \( j \) and \( k \). According to equation (4-27), the double-differenced carrier phase observables can then be written as:

\[
\nabla \Delta \phi_{j,3} = \left[ \alpha^j_1 \cdot V_{1,3} + \alpha^j_2 \cdot V_{2,3} \right] = \nabla \Delta \rho_{j,3} + \lambda \cdot \nabla \Delta N_{j,3} + \epsilon \sum_{j=1} \alpha_j \cdot \nabla \Delta \phi_j 
\]

(4-28)

\[
\nabla \Delta \phi_{k,3} = \left[ \alpha^k_1 \cdot V_{1,3} + \alpha^k_2 \cdot V_{2,3} \right] = \nabla \Delta \rho_{k,3} + \lambda \cdot \nabla \Delta N_{k,3} + \epsilon \sum_{k=1} \alpha_k \cdot \nabla \Delta \phi_k 
\]

(4-29)

Forming the linear combination between the two user stations \( j \) and \( k \) yields:

\[
\nabla \Delta \phi_{k,j} = \left[ \alpha^{k,j}_1 \cdot V_{1,3} + \alpha^{k,j}_2 \cdot V_{2,3} \right] = \nabla \Delta \rho_{k,j} + \lambda \cdot \nabla \Delta N_{k,j} + \epsilon_{k,j} 
\]

(4-30)
where $\alpha_i^{k,j}$ ($i = 1, 2$) is the difference in the $\alpha_i$ value for user stations $k$ and $j$. 

\[
\left[ \alpha_i^{k,j} \cdot V_{1,3} + \alpha_2^{k,j} \cdot V_{2,3} \right]
\]

is the ‘correction term’ for the inner baseline between these user receivers.

The advantage of equation (4-30) over equation (4-27) is the possibility of accommodating more than one user receiver operating simultaneously within the fiducial network of reference receivers. By forming the double-differenced observables between the inner (single-frequency) receivers, and using the residual vectors generated from the fiducial reference stations, the inner stations’ coordinates can be determined without the need to directly use any GPS reference station observations at all.

The procedure of generating these ‘correction terms’ for a mixed-mode GPS-based volcano deformation monitoring network can be summarised as follows. Consider a network of three fiducial reference stations equipped with dual-frequency GPS receivers surrounding an inner network of several single-frequency receivers (see Figures 2.13 and 4.7). Holding the coordinates of one fiducial site fixed, the baselines to the other two reference receivers are processed and corrections are obtained for both fiducial baselines. These are then scaled (with the help of the $\alpha$ values) according to the position of the inner receivers inside the fiducial triangle to generate double-differenced ‘correction terms’ for the inner baselines.

### 4.6 Analysis of the Empirically-Derived Correction Terms

The nature of the empirically-derived double-differenced ‘correction terms’ is investigated in this section. A range of GPS data sets were processed, incorporating a variety of baseline lengths, different geographical locations and different periods of sunspot activity (and hence varying ionospheric conditions).

#### 4.6.1 The L4 Linear Combination

In order to comment on the severity of the ionospheric delay under various conditions the difference between the L1 and L2 correction terms is formed. This linear combination
eliminates the tropospheric effect, which is assumed to be the same for both frequencies. As opposed to the L1 corrections, the remaining L1-L2 correction terms represent only the ionosphere and the negligible orbit bias (very small due to the use of precise ephemerides in this case), if the multipath effect can be ignored.

According to equation (4-24), the double-differenced observable can be written as:

$$\nabla\Delta\phi = \nabla\Delta\rho + \nabla\Delta d + \lambda \cdot \nabla\Delta N - \nabla\Delta d_{\text{ion}} + \nabla\Delta d_{\text{trop}} + \nabla\Delta d_{\text{mp}} + \epsilon_{\nabla\Delta\phi}$$

Neglecting orbit bias, multipath and observation noise, the following equations can be obtained for L1 and L2:

$$\nabla\Delta\phi_{L1} = \nabla\Delta\rho - \nabla\Delta d_{\text{ion}} + \nabla\Delta d_{\text{trop}} + \lambda_{L1} \cdot \nabla\Delta N_{L1}$$  (4-31)

$$\nabla\Delta\phi_{L2} = \nabla\Delta\rho - \frac{f_1^2}{f_2^2} \nabla\Delta d_{\text{ion}} + \nabla\Delta d_{\text{trop}} + \lambda_{L2} \cdot \nabla\Delta N_{L2}$$  (4-32)

Forming the difference between the L1 and L2 correction terms gives:

$$\nabla\Delta\phi_{L1-L2} = -\nabla\Delta d_{\text{ion}} + \lambda_{L1} \cdot \nabla\Delta N_{L1} + \frac{f_1^2}{f_2^2} \nabla\Delta d_{\text{ion}} - \lambda_{L2} \cdot \nabla\Delta N_{L2}$$

$$= \left(-1 + \frac{f_1^2}{f_2^2}\right) \cdot \nabla\Delta d_{\text{ion}} + \lambda_{L1} \cdot \nabla\Delta N_{L1} - \lambda_{L2} \cdot \nabla\Delta N_{L2}$$  (4-33)

Substituting the frequencies $f_1 = 1575.42$ MHz and $f_2 = 1227.60$ MHz for L1 and L2 yields:

$$\nabla\Delta\phi_{L1-L2} = 0.647 \cdot \nabla\Delta d_{\text{ion}} + \lambda_{L1} \cdot \nabla\Delta N_{L1} - \lambda_{L2} \cdot \nabla\Delta N_{L2}$$  (4-34)

The second and third terms on the right side of the equation are constants. Hence, under the assumptions made above, this so-called L4 linear combination represents entirely the ionospheric effect. However, the L4 corrections are muted by a factor of 0.647 and of opposite sign in comparison to the L1 corrections. The L4 observable is independent of receiver and satellite clocks and contains no receiver-satellite range term, hence it is also known as the *geometry-free* observable. While investigating the nature of the empirical
L1 correction terms, the L4 linear combination provides useful information about the magnitude of the ionospheric effect.

### 4.6.2 Analysis of Field Data Collected at Mid-Latitudes

In order to understand the nature of the double-differenced correction terms, two data sets collected in the mid-latitude region were analysed. Data for different baseline lengths were collected in Japan during a 24-hour observation period on January 2, 1997, and on March 7, 2000. While the solar activity was rather low in January 1997, it was approaching a maximum during March 2000.

In order to investigate the ionospheric effect on baselines of different lengths, under different solar activity conditions, seven baselines with lengths ranging from 26km to 101km were analysed. The GPS stations belong to the Hokkaido network, which is part of Japan’s GEONET (GPS Earth Observation Network), and are equipped with dual-frequency GPS receivers. The data were processed with a modified version of the Bernese software package to generate the empirical corrections. (In order to draw conclusions relevant to the volcano application, the following analysis is only concerned with the L1 correction terms.)

Figures 4.8 and 4.9 show the double-differenced L1 correction terms obtained for different baseline lengths for the 1997 and 2000 data sets respectively. A clear increase in ionospheric activity between the data sets is evident (see also Table 4.1 for a list of relevant parameters). Furthermore, it can be seen that ionospheric activity is mainly a daytime phenomenon in mid-latitude regions.
Fig. 4.8: L1 double-differenced correction terms for different baseline lengths

(2 January 1997)
Fig. 4.9: L1 double-differenced correction terms for different baseline lengths

(7 March 2000)
Figure 4.10 shows the standard deviation of the double-differenced L1 correction terms for different baseline lengths in January 1997 and March 2000. The effect of the increased ionospheric activity due to the solar maximum period in the year 2000 is obvious. It can be seen that the standard deviation increases more or less linearly with increasing baseline length, confirming the findings by Odijk (2000) and showing that the correct assumptions have been made. Under solar maximum conditions this trend is much more pronounced.

![Graph showing standard deviation of double-differenced correction terms for different baseline lengths (Japan)](image)

Fig. 4.10: Standard deviation of double-differenced correction terms for different baseline lengths under low (1997) and high (2000) solar activity conditions

Figure 4.11 shows the minimum and maximum correction values obtained for different baseline lengths over a period of 24 hours. It can be seen that the magnitude of the correction terms for longer baselines increases rather rapidly during solar maximum conditions. This suggests that long baselines between reference stations would not be capable of generating reliable corrections under these conditions. However, the magnitude of these biases is not entirely a function of distance, hence it is difficult to predict what should be the dimensions of the reference station network that would reliably model these distance-dependent biases.
For comparison purposes, Figure 4.12 shows the double-differenced L1 corrections for a 77km baseline over 24 hours in January 1997, and then again in March 2000. While the ionosphere remains calm, not showing much change during the 24-hour observation period in 1997, the effect of the increased ionospheric activity in 2000 can clearly be seen. Here, the diurnal variability of the ionosphere in mid-latitudes can easily be discerned. As expected, the ionosphere is most active during daylight hours between 8am and 6pm local time, and calms down after dark. Between the years 1997 and 2000 the standard deviation increased by a factor of 4.6. The magnitude of the correction terms only range from –0.232m to 0.248m in 1997, while they vary from –0.555m to 0.824m in 2000.
In order to further study the effects of the ionosphere, the L4 correction terms were formed. As mentioned earlier, the L4 corrections represent entirely the ionospheric effect, while the L1 corrections also include the tropospheric delay. Table 4.1 shows several parameters characterising both the L1 and L4 correction terms obtained for each baseline, i.e. the minimum, maximum and mean correction terms, their standard deviation (STD) and the number of double-differences involved. Figures 4.13 and 4.14 show the L4 correction terms for each 24-hour span, the sign having been reversed to enable easy comparison. It is evident that the L4 correction terms show the same signature as the L1 corrections (see Figures 4.8 and 4.9) for both data sets. This indicates that the ionosphere is indeed the dominant systematic error in the double-differenced residuals. It also confirms that multipath is not a significant effect in these data sets. The difference in magnitude of the corrections for a period of low solar activity (1997) as opposed to a solar maximum period (2000) is clearly visible.
Tab. 4.1: Correction terms for different baseline lengths

<table>
<thead>
<tr>
<th>Baseline</th>
<th>D [km]</th>
<th>min [m]</th>
<th>max [m]</th>
<th>mean [m]</th>
<th>STD [m]</th>
<th>#DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan 02.01.1997 (L1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0011-0123</td>
<td>26</td>
<td>-0.11903</td>
<td>0.18583</td>
<td>0.00003</td>
<td>0.02504</td>
<td>13795</td>
</tr>
<tr>
<td>0011-0121</td>
<td>30</td>
<td>-0.14631</td>
<td>0.12638</td>
<td>0.00480</td>
<td>0.02482</td>
<td>13828</td>
</tr>
<tr>
<td>0011-0112</td>
<td>41</td>
<td>-0.12282</td>
<td>0.16038</td>
<td>0.00275</td>
<td>0.02991</td>
<td>13732</td>
</tr>
<tr>
<td>0011-0138</td>
<td>53</td>
<td>-0.16833</td>
<td>0.24263</td>
<td>-0.00681</td>
<td>0.03226</td>
<td>13848</td>
</tr>
<tr>
<td>0007-0123</td>
<td>77</td>
<td>-0.23173</td>
<td>0.24761</td>
<td>0.01435</td>
<td>0.04722</td>
<td>13784</td>
</tr>
<tr>
<td>0007-0121</td>
<td>93</td>
<td>-0.29142</td>
<td>0.30538</td>
<td>0.01998</td>
<td>0.05845</td>
<td>13783</td>
</tr>
<tr>
<td>0007-0011</td>
<td>101</td>
<td>-0.26170</td>
<td>0.37799</td>
<td>0.01722</td>
<td>0.06470</td>
<td>13902</td>
</tr>
</tbody>
</table>

| Japan 02.01.1997 (L4 = L1-L2) | | | | | | |
| 0011-0123 | 26 | -0.07773 | 0.08481 | -0.00114 | 0.01705 | 13795 |
| 0011-0121 | 30 | -0.09202 | 0.10501 | 0.00262 | 0.01712 | 13827 |
| 0011-0112 | 41 | -0.09529 | 0.12280 | 0.00354 | 0.02151 | 13731 |
| 0011-0138 | 53 | -0.11983 | 0.15890 | -0.00324 | 0.02180 | 13848 |
| 0007-0123 | 77 | -0.15542 | 0.18877 | 0.01004 | 0.03230 | 13784 |
| 0007-0121 | 93 | -0.19970 | 0.20975 | 0.01392 | 0.03962 | 13781 |
| 0007-0011 | 101 | -0.18324 | 0.27219 | 0.01338 | 0.04119 | 13901 |

| Japan 07.03.2000 (L1) | | | | | | |
| 0011-0123 | 26 | -0.33989 | 0.23406 | -0.01376 | 0.08237 | 15067 |
| 0011-0121 | 30 | -0.43000 | 0.33577 | -0.00042 | 0.08942 | 14595 |
| 0011-0112 | 41 | -0.36978 | 0.41469 | 0.02554 | 0.11858 | 15055 |
| 0011-0138 | 53 | -0.54182 | 0.59514 | 0.00610 | 0.14531 | 15097 |
| 0007-0123 | 77 | -0.55529 | 0.82423 | 0.04445 | 0.21984 | 15033 |
| 0007-0121 | 93 | -0.64220 | 0.98978 | 0.05911 | 0.26625 | 15016 |
| 0007-0011 | 101 | -0.77948 | 1.08251 | 0.05582 | 0.29535 | 15246 |

| Japan 07.03.2000 (L4 = L1-L2) | | | | | | |
| 0011-0123 | 26 | -0.21892 | 0.19969 | -0.01075 | 0.04733 | 15067 |
| 0011-0121 | 30 | -0.28134 | 0.20743 | -0.00182 | 0.05525 | 14595 |
| 0011-0112 | 41 | -0.21345 | 0.28141 | 0.01713 | 0.07505 | 15055 |
| 0011-0138 | 53 | -0.31408 | 0.38284 | 0.00245 | 0.08926 | 15097 |
| 0007-0123 | 77 | -0.34374 | 0.53256 | 0.02866 | 0.13806 | 15033 |
| 0007-0121 | 93 | -0.40431 | 0.66483 | 0.03898 | 0.16861 | 15016 |
| 0007-0011 | 101 | -0.50554 | 0.68613 | 0.03846 | 0.17769 | 15246 |
Fig. 4.13: L4 double-differenced correction terms for different baseline lengths

(2 January 1997)
Fig. 4.14: L4 double-differenced correction terms for different baseline lengths

(7 March 2000)
Ionospheric Corrections to Improve GPS-Based Volcano Deformation Monitoring

Figure 4.15 gives an indication of how much more severe the ionospheric conditions were in 2000 relative to those in 1997. The ‘box and whisker’ plots show the median, 10th, 25th, 75th and 90th percentiles as vertical boxes with error bars. The ends of the box refer to the 25th and 75th percentiles (i.e. showing the range of the middle half of the data), while the whiskers extend to the 10th and 90th percentiles (i.e. indicating the distribution of 80% of the data). For the sake of completeness a plot referring to the L1 corrections (still including the tropospheric effect) is also shown.

![Figure 4.15: Increase in the ionospheric effect for different baseline lengths between 1997 and 2000: L4 corrections (left) and L1 corrections (right)](image)

This figure confirms that the severity of the ionospheric effect increased significantly. It also confirms that the correction terms become less reliable for longer baseline lengths. Table 4.2 tries to quantify the increase of the ionospheric conditions from 1997 to 2000. The minimum, maximum, mean and STD values of the two data sets (see Table 4.1) were compared and expressed as percentages. Some of the mean values had opposite signs and were therefore left out of the analysis.

Analysis of the L4 correction terms shows that the minimum and maximum values of the corrections increased by a factor of about 2.5, while the standard deviation increased by a factor of approximately 3.8 on average. The increase of the standard deviation shows a dependency on the baseline length, reaching a maximum factor of 4.3 for the 101km baseline. This was expected, and is in line with the trend depicted in Figures 4.10, 4.11 and 4.15. The L1 corrections give slightly higher factors.
Tab. 4.2: Percentage increase of the ionospheric effect between the 1997 and 2000 data sets in Japan

<table>
<thead>
<tr>
<th>L4 correction terms</th>
<th>L1 correction terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min (97-00)</td>
<td>Max (97-00)</td>
</tr>
<tr>
<td>Min (97-00)</td>
<td>Max (97-00)</td>
</tr>
<tr>
<td>26 km</td>
<td>282 %</td>
</tr>
<tr>
<td>30 km</td>
<td>306 %</td>
</tr>
<tr>
<td>41 km</td>
<td>224 %</td>
</tr>
<tr>
<td>53 km</td>
<td>262 %</td>
</tr>
<tr>
<td>77 km</td>
<td>221 %</td>
</tr>
<tr>
<td>93 km</td>
<td>202 %</td>
</tr>
<tr>
<td>101 km</td>
<td>276 %</td>
</tr>
<tr>
<td><strong>mean</strong></td>
<td><strong>253 %</strong></td>
</tr>
</tbody>
</table>

However, it should be stressed that these values can only be interpreted as an indication of how much more severe the ionospheric effect is during a solar maximum period compared to a period of low solar activity. Although the 1997 and 2000 data sets were collected in the same season, it is apparent that a 24-hour observation period is not enough to draw general conclusions. The high variability of the ionosphere in time and space makes it extremely difficult to derive a meaningful ‘rule-of-thumb’. (Many more data sets from different parts of the globe with longer observation periods need to be investigated in order to do this. However, this goes beyond the scope of this thesis.)

### 4.6.3 Analysis of Field Data Collected in Different Geographical Regions

The effect of the ionospheric layer on a certain baseline length as a function of geographical location was investigated. The magnitudes of the double-differenced correction terms for a 30km baseline located in mid-latitudes (Japan) and in the equatorial region (Singapore) were compared. The data were collected under solar maximum conditions on March 7, 2000, and the results are shown in Figure 4.16. The more severe ionospheric delay effects in the equatorial region are obvious. In Singapore the magnitude of the minimum and maximum corrections are −0.610m and 0.681m, respectively, with a standard deviation of 0.170m, while in Japan the values only range from −0.430m to 0.336m with a standard deviation of 0.089m. The standard deviation has
doubled while the minimum and maximum values have increased by a factor of approximately 2.

In addition, it can be seen that the ionospheric effect is mainly a daytime phenomenon in mid-latitudes. However, the graph shows that there is also a lot of ionospheric activity between local noon and sunset in Singapore. This is contrary to the expectation that most of the ionospheric activity occurs between sunset and midnight in equatorial regions, and might be due to intensified small-scale disturbances in the ionosphere during a period of increased solar activity. Furthermore, it can be identified as the primary diurnal maximum of the *equatorial anomaly*, also known as the ‘fountain effect’ (high electron concentration observed on either side of the geomagnetic equator at magnetic latitudes of around 10-20°). Huang & Cheng (1991) state that the daily equatorial anomaly generally begins to develop at around 9-10am local time, reaching its primary maximum development at 2-3pm local time. In periods of solar maximum conditions, however, the anomaly is prone to peak after sunset, and gradients in TEC are considerably larger at this secondary diurnal maximum (Skone, 2000). Horizontal gradients of up to $30\times10^{16}$ el/m$^2$ (30 TECU) have been observed in the equatorial region under solar maximum conditions.
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(Wanninger, 1993). The equatorial anomaly, as well as the additional ionospheric disturbances present around midnight in the equatorial region (Fig. 4.16), can also be clearly seen in the GIM shown in Figure 4.5.

L4 correction terms were also obtained for both baselines and are shown in Figure 4.17. The plots of the L1 and L4 corrections show the same signature, indicating that the ionosphere is indeed the main systematic error in the double-differenced residuals. Table 4.3 lists the minimum, maximum and mean corrections, their standard deviation and the number of double-differences involved for both the L1 corrections and the L4 corrections, while Table 4.4 shows the increase of ionospheric activity in terms of percentage in both cases. An increase of the ionospheric effect by a factor of about 2 is apparent. This trend is also visible in Figure 4.18.

Fig. 4.17: Double-differenced L4 corrections for a 30km baseline over 24 hours in different geographical regions (mid-latitude region and equatorial region)
Tab. 4.3: Correction terms for baselines in different geographic regions

<table>
<thead>
<tr>
<th>Baseline</th>
<th>D [km]</th>
<th>min [m]</th>
<th>max [m]</th>
<th>mean [m]</th>
<th>STD [m]</th>
<th>#DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1 corrections 07.03.2000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0011-0121</td>
<td>30</td>
<td>-0.43000</td>
<td>0.33577</td>
<td>-0.00042</td>
<td>0.08942</td>
<td>14595</td>
</tr>
<tr>
<td>NTU-CCBS</td>
<td>31</td>
<td>-0.60977</td>
<td>0.68101</td>
<td>0.02467</td>
<td>0.16976</td>
<td>16082</td>
</tr>
<tr>
<td>L4 corrections 07.03.2000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0011-0121</td>
<td>30</td>
<td>-0.28134</td>
<td>0.20743</td>
<td>-0.00182</td>
<td>0.05525</td>
<td>14595</td>
</tr>
<tr>
<td>NTU-CCBS</td>
<td>31</td>
<td>-0.34159</td>
<td>0.44913</td>
<td>0.03153</td>
<td>0.10641</td>
<td>13257</td>
</tr>
</tbody>
</table>

Tab. 4.4: Increase of the ionospheric effect for a 30km baseline in the equatorial region in comparison to mid-latitudes (March 2000)

<table>
<thead>
<tr>
<th>L4 correction terms</th>
<th>L1 correction terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>Max</td>
</tr>
<tr>
<td>121 %</td>
<td>217 %</td>
</tr>
</tbody>
</table>

Fig. 4.18: Ionospheric corrections in the mid-latitude region (Japan) and the equatorial region (Singapore): L4 corrections (left) and L1 corrections (right)

However, this statement is based on one 24-hour data set only, hence it can only give an indication of how much more severe the ionosphere is in equatorial regions compared to mid-latitude regions. Because the ionosphere is highly variable, a large amount of global data would have to be analysed in order to draw general conclusions.
4.6.4 Analysis of Field Data Collected at the Mt. Papandayan Volcano in 2000 & 2001

In early 2000 first testing of the mixed-mode GPS-based volcano deformation monitoring system took place on Gunung Papandayan in West Java, Indonesia (see Figure 2.14). Due to its close proximity to the geomagnetic equator, the ionospheric activity at the Mt. Papandayan volcano was expected to be much higher than in a mid-latitude region like Japan. Furthermore, in comparison to the data collected in Singapore, the correction terms were expected to be larger due to the much longer baseline lengths of the network surrounding Gunung Papandayan and the more pronounced effects of the equatorial anomaly in this area.

Figure 4.19 shows the double-differenced correction terms generated by the three fiducial baselines BAND-PAME (85km), BAND-GALU (72km) and GALU-PAME (60km) over 24 hours on March 7, 2000. It can be seen that the correction terms for the two baselines BAND-PAME and BAND-GALU, which were supposed to generate the corrections to be used in the data processing, are very large. The magnitude of the correction terms range from –5.556m to 4.597m (BAND-PAME) and from –2.976m to 2.830m (BAND-GALU), with standard deviations of 1.025m and 0.719m respectively. The correction terms for the third baseline range from -1.563m to 1.504m with a standard deviation of 0.377m. This confirms the hypothesis that the reliable generation of correction terms under such a scenario requires shorter baselines within the reference station network. However, it is not possible to predict a priori what the optimal (i.e. largest) spacing is. Unfortunately, an empirical process for testing reference station geometry will be necessary, employed on a case-by-case basis.

It can be seen that the maximum ionospheric effects occur between local sunset and midnight, which is as expected. However, there is also a lot of activity between 3pm and sunset. As mentioned before, this can be explained by intensified disturbances in the ionosphere caused by increased solar sunspot activity and the primary peak of the equatorial anomaly. From Figure 4.5 it is clear that this network is located in a region exposed to the greatest changes in the ionospheric conditions. Due to the high magnitude
of the correction terms and the lack of a more continuous data set, the corrections could not be reliably applied to the single-frequency data set.

Fig. 4.19: Double-differenced L1 corrections for the fiducial baselines at Mt. Papandayan, Indonesia, over 24 hours on 7 March 2000

In mid 2001 further tests were carried out on the volcano under similar ionospheric conditions. As illustrated in Figure 2.14, the fiducial baseline lengths were reduced to 53km (GUNT-PAME) and 31km (GUNT-PANG). Figure 4.20 shows the L1 correction terms obtained for these baselines for 24-hour observation periods on four consecutive days (11-14 July 2001). The highly variable nature of the ionosphere is evident over the 4-day period studied. It is hardly possible to make out a distinct trend repeating on a daily basis. Compared to Figure 4.19 the 2001 correction terms are certainly more ‘realistic’ than the values obtained in 2000. The corrections obtained for the shorter baseline seem to faithfully represent the ionospheric conditions. This is supported by the information in Table 4.5, which lists the minimum, maximum and mean L1 corrections, their standard deviation and the number of double-differences involved for both fiducial baselines on
four successive days. The standard deviation for the 31km baseline varies between 0.095m and 0.139m with the most extreme minimum and maximum corrections being -0.866m and 0.750m respectively on the fourth day. However, the results generated by the baseline GUNT-PAME indicate that a baseline length of 53km is still too long to generate reliable correction terms in the equatorial region under solar maximum conditions. As apparent in other data sets referred to earlier, high ionospheric activity is evident between local noon and sunset, which is in contrast to the expectations for this latitude band.

Tab. 4.5: Double-differenced L1 corrections for fiducial baselines at Gunung Papandayan (2001)

<table>
<thead>
<tr>
<th>Baseline</th>
<th>D [km]</th>
<th>min [m]</th>
<th>max [m]</th>
<th>mean [m]</th>
<th>STD [m]</th>
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The empirical correction terms obtained from the fiducial network around Gunung Papandayan in 2001 were applied to the single-frequency deformation monitoring network located on the volcano. The results are presented in section 5.5.
Fig. 4.20: Double-differenced L1 corrections for the fiducial baselines at Mt. Papandayan, Indonesia, on four successive days (11-14 July 2001)
4.7 Concluding Remarks

In this chapter the ionosphere and its effects on the GPS signal has been discussed. A procedure that could, in principle, be used to correct single-frequency GPS observations using empirical corrections derived from a network of dual-frequency reference receivers has been described.

A range of data sets were processed in order to investigate the nature of the empirically-derived double-differenced ‘correction terms’. The GPS data were analysed at a variety of baseline lengths, in different geographical locations and at different periods of sunspot activity (and hence varying ionospheric conditions). The following conclusions can be drawn:

- A large increase in solar activity (and hence ionospheric disturbance) is evident between the 1997 and 2000 data sets. GPS data processing in solar maximum periods is more difficult and a full correction model is necessary.
- The ionosphere is the dominant systematic error present in the ‘correction terms’.
- The standard deviation of the double-differenced ‘correction terms’ increases linearly with increasing baseline length, confirming that the correct model has been used. The rate of increase is much more severe under solar maximum conditions.
- During solar sunspot cycle maximum conditions the magnitude of the ‘correction terms’ for longer baselines in the mid-latitudes reaches several cycles. This indicates that longer baselines might not be able to generate reliable corrections under these conditions.
- Increased ionospheric activity during the daytime in mid-latitudes, and after sunset in the equatorial region, is evident. As expected, the ionospheric delay effects for sites at the equator are much larger compared to mid-latitude sites.
- The distances between the fiducial stations around the Mt. Papandayan volcano should be further reduced so as to generate reliable corrections during solar maximum conditions. Fiducial baseline lengths of below 30km seem to be necessary in order to account for the extreme ionospheric disturbances in this area. However, the optimal spacing will have to be determined empirically.