

**CONTROL OF ION MOTION IN  
ROTATING MAGNETIC FIELD  
CURRENT DRIVE**

by

**Denis C. Visentin B.Sc. (Hons)**

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Philosophy

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## DECLARATION

I hereby declare that this thesis contains no material which has been accepted for the award of any other degree or diploma in any tertiary institution, and that, to the best of knowledge and belief, it contains no material published or written by any other person except where due acknowledgement is made in the text of the thesis.

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Denis Visentin, 9<sup>th</sup> March 2007

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## ABSTRACT

This thesis presents theoretical results, a numerical model and simulation results for the control of the ion motion for a pre-formed field-reversed-configuration (FRC) using two counter-rotating magnetic fields (RMFs). One RMF (denoted the (-) RMF) is applied first to entrain the electron fluid and maintain the plasma current. In the absence of a mechanism for controlling the ion motion, if the confinement time is sufficiently large, the rotation of the ion fluid due to collisions with the electron fluid would diminish the plasma current and thus destroy the FRC equilibrium. A second RMF (denoted the (+) RMF) is applied after the (-) RMF has penetrated the plasma, to entrain the ion fluid and maintain the equilibrium.

It is shown that there exists a true steady state (the Clemente steady state), where the electron fluid rotates almost synchronously with the (-) RMF and the ion fluid rotates almost synchronously with the (+) RMF. This allows the equilibrium to be maintained indefinitely. Both RMFs penetrate much farther than a classical skin depth. The accessibility of the Clemente steady states are examined theoretically and by simulation.

A 1-D numerical model is developed to simulate the application of the RMF for two cases:

1. A constant density model where the radial motion is constrained.
2. A preformed FRC model with radial motion.

For both cases it is demonstrated that the Clemente steady states are accessible from a small class of initial conditions. The class of initial conditions may be broadened by allowing the frequency of the (+) RMF to vary. The penetration and entrainment of the (+) RMF is shown to be highly non-linear (as is well known for the (-) RMF) and hence the magnitude of the (+) RMF required for accessibility of the steady state

is much greater than that required to maintain the steady state. It is also demonstrated that it is possible to increase the closed flux of the FRC by increasing the frequency of either RMF.

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